Testing Contextuality in Cyclic Psychophysical Systems of High Ranks

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Abstract. Contextuality-by-Default (CbD) is a mathematical framework for understanding the role of context in systems with deterministic inputs and random outputs. A necessary and sufficient condition for contextuality was derived for cyclic systems with binary outcomes. In quantum physics, the cyclic systems of ranks $n = 5$, 4, and 3 are known as systems of Klyachko-type, EPR-Bell-type, and Leggett-Garg-type, respectively. In earlier publications, we examined data collected in various behavioral and social scenarios, from polls of public opinion to our own experiments with psychophysical matching. No evidence of contextuality was found in these data sets. However, those studies were confined to cyclic systems of lower ranks ($n \leq 4$). In this paper, contextuality of higher ranks ($n = 6, 8$) was tested on our data with psychophysical matching, and again, no contextuality was found. This may indicate that many if not all of the seemingly contextual effects observed in behavioral sciences are merely violations of consistent connectedness (selectiveness of influences).

Keywords: contextuality, contextuality-by-default, cyclic systems, consistent connectedness, psychophysical matching

1 Introduction

Consider a system having two external factors (or inputs) α and β, which can be deterministically manipulated, and two random outputs A and B that we interpret as responses to, or measurements of, α and β, respectively. The system can belong to any empirical domain, from quantum physics to behavioral sciences. If manipulating β does not change the marginal distribution of A and manipulating α does not change the marginal distribution of B, we say that the system is consistently connected. Physicists traditionally test contextuality by assuming consistent connectedness (referred to as "no-signaling," "no-disturbance," etc.). However, even in quantum experiments inconsistent connectedness may occur, e.g., because of context-dependent errors in measurements. In behavioral sciences inconsistent connectedness is ubiquitous. The Contextuality-by-Default (CbD) theory allows one to detect and measure contextuality, or to determine that a system is noncontextual, irrespective of whether it is consistently connected [1–8]. In quantum physics, many experiments and theoretical considerations demonstrate the existence of contextual systems [9–15], including in cases when consistent connectedness is violated [8]. By contrast, we found no evidence of contextuality in various social and behavioral data sets, from polls of public opinion to visual illusions to conjoint choices to word combinations to psychophysical matching [16, 17].

Most of the experimental studies of contextuality, both in quantum physics and in behavioral and social sciences, have been confined to cyclic systems [4,6,8], in which each entity being measured or responded to enters in two contexts and each context contains exactly two entities. In this paper we only deal with cyclic systems of even ranks, those that can be formed using the paradigm with two experimental factors (or inputs) α, β and two outputs in response to the two factors. A cyclic system of an even rank $2n \geq 4$ can be extracted from a design in which α and β vary on n levels each, denoted $\alpha_1, \alpha_2, \ldots, \alpha_n$ and $\beta_1, \beta_2, \ldots, \beta_n$. Out of n^2 possible treatments one extracts $2n$ pairs, we call contexts, whose elements form a cycle, e.g.,

$$\begin{array}{cccccc} \text{Context 1} & \text{Context 2} & \ldots & \text{Context } (2n-1) & \text{Context } 2n \\ (\alpha_1, \beta_1) & (\beta_1, \alpha_2) & \ldots, & (\alpha_n, \beta_n) & (\beta_n, \alpha_1) \end{array}. \tag{1}$$

This is a cyclic system of rank $2n$. The outputs of the system corresponding to these $2n$ contexts are $2n$ pairs of random variables

$$(A_{11}, B_{11}), (B_{21}, A_{21}), \ldots, (A_{nn}, B_{nn}), (B_{1n}, A_{1n}), \tag{2}$$

where A_{ij} is interpreted as a response to (measurement of) α_i in the context (α_i, β_j), and B_{ij} is interpreted as a response to (measurement of) β_j in the same context, where $i, j \in \{1, \ldots, n\}$. It is assumed in addition that each random output is binary, with values denoted $-1, +1$. The random variables A_{ij} and B_{ij} (recorded in the same context) are jointly distributed, so that, e.g., the joint probability of $A_{ij} = 1$ and $B_{ij} = -1$ is well-defined. However, according to CbD, any two random outputs recorded in different contexts, such as A_{ij} and $B_{i'j'}$ or A_{ij} and $A_{i'j'}$, with $(i, j) \neq (i', j')$, are stochastically unrelated, have no joint distribution [3–5, 7, 8, 16].

In CbD, the system just described is considered noncontextual if and only if the $2n$ pairs of random variables in (2) can be coupled (imposed a joint distribution on) so that any two random variables responding to the same factor point in different contexts (i.e., A_{ij} and $A_{ij'}$, or B_{ij} and $B_{i'j}$) are equal to each other with maximal possible probability, given their individual distributions [4,5,7,8,16,17]. A necessary and sufficient condition for noncontextuality of a cyclic system (2) was derived in Refs. [4, 6]:

$$\begin{aligned} \mathit{\Delta} C = \mathrm{s}_1 \left(\langle A_{11} B_{11} \rangle, \langle B_{21} A_{21} \rangle, \ldots, \langle A_{nn} B_{nn} \rangle, \langle B_{1n} A_{1n} \rangle \right) \\ - \mathsf{ICC} - (2n-2) \leq 0, \end{aligned} \tag{3}$$

where $\langle\cdot\rangle$ denotes expected value, $\mathrm{s}_1(x_1,\ldots,x_k)$ is the maximum of all linear combinations $\pm x_1 \pm \ldots \pm x_k$ with odd numbers of minuses, and

$$\begin{aligned}\mathsf{ICC} &= |\langle A_{11}\rangle - \langle A_{1n}\rangle| + |\langle B_{11}\rangle - \langle B_{21}\rangle| \\ &+ \ldots + \left|\left\langle A_{n(n-1)}\right\rangle - \langle A_{nn}\rangle\right| + |\langle B_{nn}\rangle - \langle B_{1n}\rangle| \,.\end{aligned} \tag{4}$$

If a system is consistently connected, ICC vanishes.

Experimental studies of cyclic systems in quantum physics were confined to ranks 3, 4, and 5 (see Refs. [4, 8] for an overview). In behavioral and social experiments and surveys the ranks of the cyclic systems explored were 2, 3, and 4 (see Refs. [16, 17] for an overview). In the present study, we analyze cyclic systems of ranks 4, 6, and 8.

2 Experiments

The experimental design and procedure were described in detail in Ref. [17]. Three different psychophysical matching tasks were used (Figure 1): dot position reproduction task (Experiment 1(a) and 1(b)), concentric circles reproduction task (Experiment 2(a), 2(b) and 2(c)), and floral shape reproduction task (Experiment 3(a) and 3(b)). Each of the seven experiments was conducted on three participants.

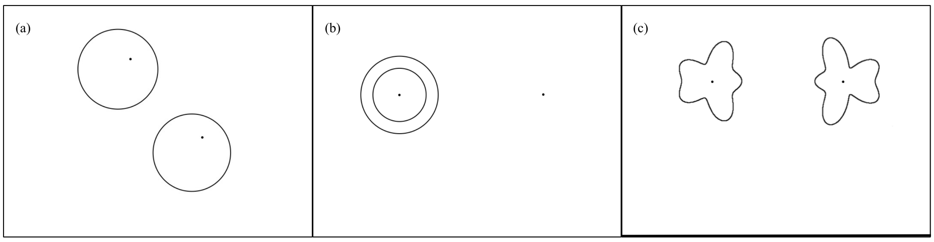

Fig. 1. Stimuli used in the (a) dot position reproduction task, (b) concentric circles reproduction task, and (c) floral shape reproduction task.

In each experimental trial, the participants were shown two stimuli on a computer screen, as shown in Fig. 1. One was a fixed stimulus, the other stimulus was adjustable, by means of rotating a trackball. The participants were required to change this stimulus until it appeared to match the position or shape of the fixed target stimulus. Once a match was achieved, she or he clicked the button on the trackball to terminate the trial. Each stimulus was characterized by two parameters. For the target stimulus these parameters are denoted as α and β, and their values in each trial were generated from a pre-defined set of numbers. The values of the same parameters in the matching stimulus are denoted A

and B (as they randomly vary for given values of α and β). Table 1 shows the parameters used.

The trials were separated by .5 second intervals. Each experiment took several days, each of which consisted of about 200 trials with a break in the middle. Each such session began by a practice series of 10 trials (which were not used for data analysis).

The original data sets for all the experiments are available as Excel files online (http://dx.doi.org/10.7910/DVN/OJZKKP). Each file corresponds to one participant in one experiment.

2.1 Participants

All the participants were students at Purdue University. The first author of this paper, labeled as P3, participated in all the experiments. Participants P1 and P2 participated in Experiments 1(a) and 2(a), and Participants P4 and P5 in Experiments 1(b), 2(b), 2(c), 3(a), and 3(b). All participants were about 25 years old and had normal or corrected to normal vision.

Table 1. External factors (α, β) and random outputs (A, B) for the three types of tasks.

Task	α	β	A	B
Dot position reproduction (rectangular coordinates)	Horizontal coordinate of the target dot	Vertical coordinate of the target dot	Horizontal coordinate of the matching dot	Vertical coordinate of the matching dot
Dot position reproduction (polar coordinates)	Radial coordinate of the target dot	Angular coordinate of the target dot	Radial coordinate of the matching dot	Angular coordinate of the matching dot
Concentric circle reproduction	Radius of the target circle 1	Radius of the target circle 2	Radius of the matching circle 1	Radius of the matching circle 2
Floral shape reproduction, see (5)	Amplitude 1 of the target shape	Amplitude 2 of the target shape	Amplitude 1 of the matching shape	Amplitude 2 of the matching shape

2.2 Stimuli and Procedure

Visual stimuli consisting of curves and dots were presented on a flat-panel monitor. The diameter of the dots and the width of the curves was 5 pixels (px). The stimuli were grayish-white on a comfortably low intensity background. The participants viewed the stimuli in darkness using a chin rest with a forehead support at the distance of 90 cm from the monitor, making 1 screen pixel approximately 62 sec arc.

Experiment 1 In Experiment 1(a), each trial began with presenting two circles with a dot in the first quadrant of each circle (as shown in Figure 1(a)). The dot in the upper left circle was fixed at one of randomly chosen six positions. These six positions contained a 2×2 "rectangular" sub-design: $\{32$ px, 64 px$\} \times \{32$ px, 64 px$\}$ and a 2×2 "polar" sub-design: $\{53.67$ px, 71.55 px$\} \times \{63.43$ deg, 26.57 deg$\}$. The coordinates were recorded using the center of the circle as the origin. The position adjustable dot was in the bottom right circle. The task was to move the bottom right dot by rotating the trackball to a position that matched that of the fixed one. There were 1200 trials overall.

Experiment 1(b) was identical to Experiment 1(a) except the horizontal coordinate and vertical coordinate of the target dot were random integers drawn from the interval [20 px, 80 px). This "rectangular" design also contained a "polar" sub-design [40 px, 90 px)$\times$[30 deg, 60 deg). The overall number of trials for the "rectangular" design was 1800, for the polar sub-design about 900.

Experiment 2 In each trial of Experiment 2(a), the target stimulus on the left consisted of two concentric circles and a dot in their center. The radii of circle 1 and circle 2 were randomly chosen from the sets $\{16$ px, 56 px, 64 px$\}$ and $\{48$ px, 72 px, 80 px$\}$, respectively. At the beginning of each trial the right stimulus was a dot. The participants had to reproduce the target stimulus by rotating the trackball to "blow up" two circles from that dot one by one. They had the freedom to produce the inner or the outer circle first. Once the first matching circle was produced, the participants clicked a button on the trackball to confirm this circle and then the program enabled them to "blow up" the other circle. After the second circle was created, the trial was terminated by clicking the same button. There were 1800 trials overall.

Experiment 2(b) was identical to Experiment 2(a) except that in each trial the radii of the target circles were randomly chosen from four possibilities $\{12$ px, 24 px$\} \times \{18$ px, 30 px$\}$. There were 1600 trials overall.

Experiment 2(c) was identical to Experiment 2(a) except that in each trial the radii of the target circles were numbers randomly chosen from [18 px, 48 px) $\times$[56 px, 86 px). There were 1800 trials overall.

Experiment 3 Two floral shapes (Figure 1(c)) were presented simultaneously in each trial in Experiment 3(a). The target one was on the left. The right one was modifiable. Each floral shape was generated by a function

$$\begin{aligned} x &= \cos(.02\pi\Delta)[70 + \alpha\cos(.06\pi\Delta) + \beta\cos(.1\pi\Delta)], \\ y &= \sin(.02\pi\Delta)[70 + \alpha\cos(.06\pi\Delta) + \beta\cos(.1\pi\Delta)], \end{aligned} \tag{5}$$

where Δ is polar angle and x and y are the horizontal and vertical coordinates (in pixels). For a matching floral shape, α, β are replaced with A, B, respectively. The amplitudes α, β of the target shape were randomly chosen from the sets {-18 px, 10 px, 14 px} and {-16 px, -12 px, 20 px}, respectively. The two amplitudes of the right shape were randomly initialized from the interval [-35 px, 35 px). The participants were asked to match the left shape by modifying the right shape by rotating the trackball. There were 1800 trials overall.

Experiment 3(b) was identical to Experiment 3(a) except that the two amplitudes of the target shape were randomly chosen numbers from the interval [-30 px, 30 px).

3 Results

In each experiment the matching points that were too far from the target values were considered outliers and they were removed from data analysis. The outliers made less than 1% of all data. Ref. [17] briefly reported how contextuality for cyclic systems of rank 4 was tested using the data collected from our seven experiments. In this paper, we present the contextuality test for rank 4 in greater details, and add the analyses for cyclic systems of ranks 6 and 8, using the same data.

3.1 Testing Contextuality for Rank 4

A cyclic system of rank 4 can be represented by four contexts

$$\begin{array}{cccc} \text{Context 1} & \text{Context 2} & \text{Context 3} & \text{Context 4} \\ (\alpha_1, \beta_1) & (\beta_1, \alpha_2) & (\alpha_2, \beta_2) & (\beta_2, \alpha_1) \end{array}. \tag{6}$$

To form such a system, we chose $\{\alpha_1, \alpha_2\} \times \{\beta_1, \beta_2\} = \{32\,\text{px}, 64\,\text{px}\} \times \{32\,\text{px}, 64\,\text{px}\}$ for the "rectangular" sub-design of Experiment 1(a). The "polar" sub-designs of Experiment 1(a) and Experiment 2(b) also have 2×2 structures, and they were presented as cyclic systems analogously. Experiment 2(a) and Experiment 3(a) have 3×3 factorial designs. We extracted 9 cyclic systems of rank 4 from each of them by selecting two α's and two β's from the sets of α and β. The "rectangular" design of Experiment 1(b) and the "polar" sub-designs of Experiment 1(b), Experiment 2(c), and Experiment 3(b) have external factors spanning certain intervals. In order to have a cyclic system of rank 4, each interval was dichotomized into two subintervals. For instance, four experimental conditions $(\alpha_{i_1}, \beta_{i_2})$, $i_1, i_2 \in \{1, 2\}$, are formed in the "rectangular" design of Experiment 1(b) if one chooses $\alpha_1 = [20\,\text{px}, 50\,\text{px})$, $\alpha_2 = [50\,\text{px}, 80\,\text{px})$, $\beta_1 = [20\,\text{px}, 50\,\text{px})$, and $\beta_2 = [50\,\text{px}, 80\,\text{px})$. Of course other cut-off points can be chosen to dichotomize the intervals. In this paper, we only report the results from the midpoint-dichotomized data sets.

Irrespective of the experiment, the random outputs $A_{i_1 i_2}, B_{i_1 i_2}$ should each be dichotomized. The two values for each random variable were defined by choosing a value a_{i_1} and a value b_{i_2} and computing

$$A^*_{1i_2} = \begin{cases} +1 \text{ if } A_{1i_2} > a_1 \\ -1 \text{ if } A_{1i_2} \le a_1 \end{cases}, \quad A^*_{2i_2} = \begin{cases} +1 \text{ if } A_{2i_2} > a_2 \\ -1 \text{ if } A_{2i_2} \le a_2 \end{cases},$$
$$B^*_{i_1 1} = \begin{cases} +1 \text{ if } B_{i_1 1} > b_1 \\ -1 \text{ if } B_{i_1 1} \le b_1 \end{cases}, \quad B^*_{i_1 2} = \begin{cases} +1 \text{ if } B_{i_1 2} > b_2 \\ -1 \text{ if } B_{i_1 2} \le b_2 \end{cases}. \tag{7}$$

We chose a value a_1 as any integer (in pixels) between max(minA_{11}, minA_{12}) and min(maxA_{11}, maxA_{12}), b_1 as any integer (in pixels or degrees) between max(minB_{11}, minB_{21}) and min(maxB_{11}, maxB_{21}), and analogously for a_2 and b_2. The total number of the rank-4 systems thus formed varied from 3024 to 11,663,568 per experiment per participant. For each choice of the quadruple, we applied the test (3)-(4) to the distributions of the obtained A^* and B^* variables. No positive $\triangle C$ was observed, indicating the absence of contextuality in the rank 4 cyclic system for each participant in each experiment.

We present an example to illustrate how the test of (non)contextuality was conducted. For participant P3 in the "polar" sub-design of Experiment 1(a), one choice of the quadruple was $(a_1, a_2, b_1, b_2) = (72 \text{ px}, 67 \text{ px}, 60 \text{ deg}, 23 \text{ deg})$. The distributions of the random outputs for the four contexts indexed as in (6) are presented in Table 2, where the numbers in the grids are joint probabilities and the numbers outside are marginal probabilities.

Table 2. Distributions of the random outputs for the cyclic system of rank 4, P3 in the "polar" sub-design of Experiment 1(a).

Context 1	$B_{11} > b_1$	$B_{11} \le b_1$	
$A_{11} > a_1$	.0056	0	.0056
$A_{11} \le a_1$	.3944	.6	.9944
	.4	.6	

Context 2	$B_{21} > b_1$	$B_{21} \le b_1$	
$A_{21} > a_2$	.6403	.3399	.9802
$A_{21} \le a_2$	.0099	.0099	.0198
	.6502	.3498	

Context 3	$B_{22} > b_2$	$B_{22} \le b_2$	
$A_{22} > a_2$	.5789	.4167	.9956
$A_{22} \le a_2$	.0044	0	.0044
	.5833	.4167	

Context 4	$B_{12} > b_2$	$B_{12} \le b_2$	
$A_{12} > a_1$	.0273	.0219	.0492
$A_{12} \le a_1$	.4699	.4809	.9508
	.4972	.5028	

We have, in reference to (3)-(4)

$$\mathrm{s}_1 \left(\langle A^*_{11} B^*_{11} \rangle, \langle B^*_{21} A^*_{21} \rangle, \langle A^*_{22} B^*_{22} \rangle, \langle B^*_{12} A^*_{12} \rangle \right) = \mathrm{s}_1 (.2112, .3004, .1578, .0164) = 0.653,$$

$$\begin{aligned}\mathsf{ICC} &= |\langle A^*_{11}\rangle - \langle A^*_{12}\rangle| + |\langle B^*_{11}\rangle - \langle B^*_{21}\rangle| + |\langle A^*_{21}\rangle - \langle A^*_{22}\rangle| + |\langle B^*_{22}\rangle - \langle B^*_{12}\rangle| \\ &= |(-.9016) - (-.9888)| + |(-.2) - .3004| + |.9604 - .9912| + |.1666 - (-.0056)| \\ &= .7906.\end{aligned}$$

With $2n - 2 = 4 - 2 = 2$ we obtain

$$\varDelta C = -2.1376 < 0,$$

no evidence of contextuality.

3.2 Testing Contextuality for Rank 6

Both Experiment 2(a) and Experiment 3(a) have 3×3 designs: $\{\alpha_1, \alpha_2, \alpha_3\} \times \{\beta_1, \beta_2, \beta_3\}$. From each of them we extracted one cyclic system of rank 6,

$$\begin{array}{cccccc} \text{Context 1} & \text{Context 2} & \text{Context 3} & \text{Context 4} & \text{Context 5} & \text{Context 6} \\ (\alpha_1, \beta_1) & (\beta_1, \alpha_2) & (\alpha_2, \beta_2) & (\beta_2, \alpha_3) & (\alpha_3, \beta_3) & (\beta_3, \alpha_1) \end{array}, \tag{8}$$

and labeled the random outputs A, B accordingly.

The "rectangular" design of Experiment 1(b), the "polar" sub-designs of Experiment 1(b), Experiment 2(c), and Experiment 3(b) are the systems with quasi-continuous factors. These factors were discretized into three levels by using the one-third quantile and the two-third quantile of each interval as cut-off points.

Again, the random outputs should be dichotomized in each experiment. We chose a value a_{i_1} and a value b_{i_2}, $i_1, i_2 \in \{1, 2, 3\}$, and defined

$$\begin{array}{ll} A^*_{1i_2} = \begin{cases} +1 \text{ if } A_{1i_2} > a_1 \\ -1 \text{ if } A_{1i_2} \leq a_1 \end{cases}, & A^*_{2i_2} = \begin{cases} +1 \text{ if } A_{2i_2} > a_2 \\ -1 \text{ if } A_{2i_2} \leq a_2 \end{cases}, \\ A^*_{3i_2} = \begin{cases} +1 \text{ if } A_{3i_2} > a_3 \\ -1 \text{ if } A_{3i_2} \leq a_3 \end{cases}, & B^*_{i_1 1} = \begin{cases} +1 \text{ if } B_{i_1 1} > b_1 \\ -1 \text{ if } B_{i_1 1} \leq b_1 \end{cases}, \\ B^*_{i_1 2} = \begin{cases} +1 \text{ if } B_{i_1 2} > b_2 \\ -1 \text{ if } B_{i_1 2} \leq b_2 \end{cases}, & B^*_{i_1 3} = \begin{cases} +1 \text{ if } B_{i_1 3} > b_3 \\ -1 \text{ if } B_{i_1 3} \leq b_3 \end{cases}. \end{array} \tag{9}$$

We chose a_1 as any integer between max(minA_{11}, minA_{13}) and min(maxA_{11}, maxA_{13}), b_1 as any integer between max(minB_{11}, minB_{21}) and min(maxB_{11}, maxB_{21}), and analogously for a_2, a_3, b_2, and b_3 for the experiments with discrete factor points (Experiment 2(a) and Experiment 3(a)). For the experiments with quasi-continuous factors, we chose $a_1, a_2, a_3, b_1, b_2, b_3$ as every third integer within the corresponding range . The total number of the rank-6 systems thus formed varied from 18,000 to 31,905,600 per experiment per participant. For each such choice of the sextuple $(a_1, a_2, a_3, b_1, b_2, b_3)$ we conducted the test (3)-(4). No positive $\triangle C$ was observed for the systems of rank 6 we investigated.

We present an example of how the test (3)-(4) was conducted. For participant P1 in Experiment 2(a), in which $\{\alpha_1, \alpha_2, \alpha_3\} \times \{\beta_1, \beta_2, \beta_3\} = \{$16 px, 56 px, 64 px$\} \times \{$48 px, 72 px, 80 px$\}$, one choice of the sextuple was $(a_1, a_2, a_3, b_1, b_2, b_3) =$ (16 px, 56 px, 64 px, 48 px, 72 px, 80 px). The distributions of the random outputs for the six contexts indexed as in (8) are presented in Table 3.

We have

$$\begin{array}{c} \mathrm{s}_1\left(\langle A^*_{11}B^*_{11}\rangle, \langle B^*_{21}A^*_{21}\rangle, \langle A^*_{22}B^*_{22}\rangle, \langle B^*_{32}A^*_{32}\rangle, \langle A^*_{33}B^*_{33}\rangle, \langle B^*_{13}A^*_{13}\rangle\right) \\ = \mathrm{s}_1\left(.1192, .4843, .5116, .4865, .8246, .2736\right) = 2.4613, \end{array}$$

Table 3. Distributions of the random outputs for the cyclic system of rank 6, P1 in Experiment 2(a).

Context 1	$B_{11} > b_1$	$B_{11} \leq b_1$	
$A_{11} > a_1$	.2124	.2487	.4611
$A_{11} \leq a_1$	.1917	.3472	.5389
	.4041	.5959	

Context 2	$B_{21} > b_1$	$B_{21} \leq b_1$	
$A_{21} > a_2$	.2353	.1041	.3394
$A_{21} \leq a_2$	.1538	.5068	.6606
	.3891	.6109	

Context 3	$B_{22} > b_2$	$B_{22} \leq b_2$	
$A_{22} > a_2$	.1221	.0814	.2035
$A_{22} \leq a_2$	.1628	.6337	.7965
	.2849	.7151	

Context 4	$B_{32} > b_2$	$B_{32} \leq b_2$	
$A_{32} > a_3$	.2703	.0586	.3288
$A_{32} \leq a_3$	.1982	.4730	.6712
	.4685	.5316	

Context 5	$B_{33} > b_3$	$B_{33} \leq b_3$	
$A_{33} > a_3$	.0702	.0468	.1170
$A_{33} \leq a_3$	.0409	.8421	.8830
	.1111	.8889	

Context 6	$B_{13} > b_3$	$B_{13} \leq b_3$	
$A_{13} > a_1$	.1321	.1981	.3302
$A_{13} \leq a_1$	.1651	.5047	.6698
	.2972	.7028	

$$\begin{aligned}
\mathsf{ICC} =& |\langle A^*_{13}\rangle - \langle A^*_{11}\rangle| + |\langle B^*_{11}\rangle - \langle B^*_{21}\rangle| + |\langle A^*_{22}\rangle - \langle A^*_{21}\rangle| + |\langle B^*_{22}\rangle - \langle B^*_{32}\rangle| \\
&+ |\langle A^*_{32}\rangle - \langle A^*_{33}\rangle| + |\langle B^*_{33}\rangle - \langle B^*_{13}\rangle| \\
=& |-.0778 - (-.3396)| + |-.1918 - (-.2218)| + |-.3212 - (-.593)| \\
&+ |-.4302 - (-.0632)| + |-.3424 - (-.7660)| + |-.7778 - (-.4056)| \\
=& .2618 + 0.03 + .2718 + .367 + .4236 + .3722 = 1.7264,
\end{aligned}$$

whence

$$\Delta C = 2.4613 - 1.7264 - (6 - 4) = -3.2651 < 0,$$

no evidence of contextuality.

3.3 Testing Contextuality for Rank 8

The "rectangular" design of Experiment 1(b), the "polar" sub-designs of Experiment 1(b), Experiment 2(c), and Experiment 3(b) have quasi-continuous factors. Each factor in each experiment was discretized into four levels in order to form a rank 8 cyclic system. Three points should be chosen for each factor to make this discretization. We chose the first quartile point, the second quartile (median) point, and the third quartile point of each interval. A cyclic system of rank 8 was extracted from each experiment:

$$\begin{array}{cccccccc}
\text{Context 1} & \text{Context 2} & \text{Context 3} & \text{Context 4} & \text{Context 5} & \text{Context 6} & \text{Context 7} & \text{Context 8} \\
(\alpha_1, \beta_1) & (\beta_1, \alpha_2) & (\alpha_2, \beta_2) & (\beta_2, \alpha_3) & (\alpha_3, \beta_3) & (\beta_3, \alpha_4) & (\alpha_4, \beta_4) & (\beta_4, \alpha_1)
\end{array}, \quad (10)$$

with the random outputs labeled accordingly.

To dichotomize the outputs, we chose a value a_{i_1} and a value b_{i_2}, $i_1, i_2 \in \{1, 2, 3, 4\}$ to define

$$A^*_{1i_2} = \begin{cases} +1 \text{ if } A_{1i_2} > a_1 \\ -1 \text{ if } A_{1i_2} \leq a_1 \end{cases}, \quad A^*_{2i_2} = \begin{cases} +1 \text{ if } A_{2i_2} > a_2 \\ -1 \text{ if } A_{2i_2} \leq a_2 \end{cases},$$
$$A^*_{3i_2} = \begin{cases} +1 \text{ if } A_{3i_2} > a_3 \\ -1 \text{ if } A_{3i_2} \leq a_3 \end{cases}, \quad A^*_{4i_2} = \begin{cases} +1 \text{ if } A_{4i_2} > a_4 \\ -1 \text{ if } A_{4i_2} \leq a_4 \end{cases},$$
$$B^*_{i_11} = \begin{cases} +1 \text{ if } B_{i_11} > b_1 \\ -1 \text{ if } B_{i_11} \leq b_1 \end{cases}, \quad B^*_{i_12} = \begin{cases} +1 \text{ if } B_{i_12} > b_2 \\ -1 \text{ if } B_{i_12} \leq b_2 \end{cases},$$
$$B^*_{i_13} = \begin{cases} +1 \text{ if } B_{i_13} > b_3 \\ -1 \text{ if } B_{i_13} \leq b_3 \end{cases}, \quad B^*_{i_14} = \begin{cases} +1 \text{ if } B_{i_14} > b_4 \\ -1 \text{ if } B_{i_14} \leq b_4 \end{cases}. \tag{11}$$

For each rank 8 cyclic system, we chose a_1 as every sixth integer between max(minA_{11}, minA_{14}) and min(maxA_{11}, maxA_{14}), b_1 as every sixth integer between max(minB_{11}, minB_{21}) and min(maxB_{11}, maxB_{21}), and analogously for a_2, a_3, a_4, b_2, b_3, and b_4. The total number of the rank-8 systems thus formed varied from 432 to 6,453,888 per experiment per participant. For each thus obtained octuple we conducted the test (3)-(4). No positive $\triangle C$ was observed in all the investigated cyclic systems of rank 8.

To give an example, for participant P4 in Experiment 3(b), one choice of the octuple was $(a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4)$ = (-21 px, -6 px, 6 px, 21 px, -21 px, -9 px, 9 px, 21 px). The distributions of the random outputs for the eight contexts indexed as in (10) are presented in Table 4.

We have then

$$\mathrm{s}_1 \left(\langle A^*_{11} B^*_{11} \rangle, \langle B^*_{21} A^*_{21} \rangle, \langle A^*_{22} B^*_{22} \rangle, \langle B^*_{32} A^*_{32} \rangle, \langle A^*_{33} B^*_{33} \rangle, \langle B^*_{43} A^*_{43} \rangle, \langle A^*_{44} B^*_{44} \rangle, \langle B^*_{14} A^*_{14} \rangle \right)$$
$$= \mathrm{s}_1 (.0644, .0857, .0518, .2608, .0377, .0476, .0746, .0096) = .613,$$

$$\begin{aligned}
\mathsf{ICC} = & |\langle A^*_{11} \rangle - \langle A^*_{14} \rangle| + |\langle B^*_{11} \rangle - \langle B^*_{21} \rangle| + |\langle A^*_{22} \rangle - \langle A^*_{21} \rangle| + |\langle B^*_{22} \rangle - \langle B^*_{32} \rangle| \\
& + |\langle A^*_{33} \rangle - \langle A^*_{32} \rangle| + |\langle B^*_{43} \rangle - \langle B^*_{33} \rangle| + |\langle A^*_{44} \rangle - \langle A^*_{43} \rangle| + |\langle B^*_{14} \rangle - \langle B^*_{44} \rangle| \\
= & |-.129 - (-.162)| + |-.3226 - (-.2952)| + |-.0172 - (-.1428)| + |.069 - .2174| \\
& + |.1888 - .1738| + |-.1746 - (-.132)| + |.2686 - (.1110)| + |-.2 - (-.0596)| \\
= & .033 + .0274 + .1256 + .1484 + .015 + .0426 + .1576 + .1404 = .6902,
\end{aligned}$$

whence

$$\varDelta C = .613 - .6902 - (8 - 2) = -6.0772 < 0,$$

no evidence of contextuality.

4 Conclusions

Contextuality-by-default is a mathematical framework that allows to classify systems as contextual or noncontextual. Experimental data suggest that the

Table 4. Distributions of the random outputs for the cyclic system of rank 8, P4 in Experiment 3(b).

Context 1	$B_{11} > b_1$	$B_{11} \leq b_1$	
$A_{11} > a_1$	.1532	.2823	.4355
$A_{11} \leq a_1$	.1855	.3790	.5645
	.3387	.6613	

Context 2	$B_{21} > b_1$	$B_{21} \leq b_1$	
$A_{21} > a_2$	.1619	.2667	.4286
$A_{21} \leq a_2$	.1905	.3810	.5715
	.3524	.6477	

Context 3	$B_{22} > b_2$	$B_{22} \leq b_2$	
$A_{22} > a_2$	.2759	.2155	.4914
$A_{22} \leq a_2$	.2586	.2500	.5086
	.5345	.4655	

Context 4	$B_{32} > b_2$	$B_{32} \leq b_2$	
$A_{32} > a_3$	.4130	.1739	.5869
$A_{32} \leq a_3$	.1957	.2174	.4131
	.6087	.3913	

Context 5	$B_{33} > b_3$	$B_{33} \leq b_3$	
$A_{33} > a_3$	.2736	.3208	.5944
$A_{33} \leq a_3$	.1604	.2453	.4057
	.4340	.5661	

Context 6	$B_{43} > b_3$	$B_{43} \leq b_3$	
$A_{43} > a_4$	.2460	.3095	.5555
$A_{43} \leq a_4$	.1667	.2778	.4445
	.4127	.5873	

Context 7	$B_{44} > b_4$	$B_{44} \leq b_4$	
$A_{44} > a_4$	.3209	.3134	.6343
$A_{44} \leq a_4$	.1493	.2164	.3657
	.4702	.5298	

Context 8	$B_{14} > b_4$	$B_{14} \leq b_4$	
$A_{14} > a_1$	.1619	.2571	.4190
$A_{14} \leq a_1$	.2381	.3429	.5810
	.4	.6	

noncontextuality boundaries are generally breached in quantum physics [8]. In Refs. [16, 17] we reviewed several behavioral and social scenarios to conclude that none of them provided evidence for contextuality. By examining the psychophysical data collected in our laboratory, we found no contextuality for cyclic systems of different ranks, including high ranks (6 and 8) that have never been analyzed before. We suspect that it may be generally true that human and social behaviors are not contextual in the same sense in which quantum systems are.

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