Introduction to Statistics in Psychology
PSY 201
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Lecture 12
probability
Why casinos make money.

DESCRIPTIVE STATISTICS
most of what we have discussed so far
is called descriptive statistics
• distributions
• graphs
• central tendency
• variation
• correlation
describe sets of data

INFERENTIAL STATISTICS
given a set of data from a sample
we want to infer something about the entire population
• mean
• standard deviation
• correlation
• ...
never with certainty
with probability

PROBABILITY
number between 0 and 1
probability of event A is written as
\[ P(A) \]
if
\[ P(A) = 1.0 \]
it indicates with certainty that event A will happen
if
\[ P(A) = 0 \]
it indicates with certainty that event A will not happen

PROBABILITY LAWS
there are specific rules to probability
we want to know the probability of many events, pairs of events, contingent events, ...
how to calculate probabilities depends upon
• Complements
• Mutually exclusive compound events
• Nonmutually exclusive events
• Statistically independent joint events
• Statistically dependent joint events

SINGLE EVENTS
precise definition requires high-level mathematics
intuitive definition is that probability of a single event is the ratio of the number of possible outcomes that include the event to the total number of possible outcomes
\[
P(a \text{ die coming up 3}) = \frac{\text{Number of outcomes that include 3}}{\text{Total number of outcomes}}
\]
\[
P(a \text{ die coming up 3}) = \frac{1}{6} \approx 0.167
\]
1 2 3 4 5 6
COMPLEMENTS

suppose we know the probability \( P(A) \), where \( A \) is some event

then if \( \overline{A} \) represents “not \( A \)” (called the complement of \( A \))

\[
P(\overline{A}) = 1 - P(A)
\]

when \( A \) = turning up a 3 on a die

\( \overline{A} \) means turning up anything other than a 3

since \( P(A) = 0.167 \)

\[
P(\overline{A}) = 1.0 - 0.167 = 0.833
\]

\[
1 2 3 4 5 6
\]

COMPOUND EVENTS

sometimes we know the probability of two events \( A \) and \( B \)

and we want to know the probability of event \( A \) or \( B \)

\[
P(\text{turning up a } 3 \text{ or a } 4 \text{ on a die})
\]

these are mutually exclusive events

one or the other

\[
P(A \text{ or } B) = P(A) + P(B)
\]

MUTUALLY EXCLUSIVE

for mutually exclusive compound events, calculating the probability of the compound is easy

consider probability of rolling numbers on a die

\[
P(\text{a 3 or a 4}) = P(3) + P(4)
\]

\[
P(\text{turning up a 3 or a 4 on a die}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}
\]

\[
1 2 3 4 5 6
\]

in general, if \( A \) and \( B \) are mutually exclusive

\[
P(A \text{ or } B) = P(A) + P(B)
\]

NONMUTUALLY EXCLUSIVE

sometimes events are not mutually exclusive

\[
P(\text{number } \leq 3 \text{ or odd}) = P(\text{number } \leq 3) + P(\text{odd}) - P(\text{number } \leq 3 \text{ and odd})
\]

\[
= \frac{1}{2} + \frac{1}{2} - \frac{3}{6} = \frac{1}{6} + \frac{3}{6} = \frac{2}{6} = \frac{1}{3}
\]

in general

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)
\]

when the events are mutually exclusive, \( P(A \text{ and } B) = 0 \)

and we get the rule for mutually exclusive events

\[
P(A \text{ or } B) = P(A) + P(B)
\]

NONMUTUALLY EXCLUSIVE

subtract out common probability

\[
P(\text{number } \leq 3 \text{ or odd}) = P(\text{number } \leq 3) + P(\text{odd}) - P(\text{number } \leq 3 \text{ and odd})
\]

\[
= \frac{1}{2} + \frac{1}{2} - \frac{3}{6} = \frac{1}{6} + \frac{3}{6} = \frac{2}{6} = \frac{1}{3}
\]

in general

\[
P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)
\]

when the events are mutually exclusive, \( P(A \text{ and } B) = 0 \)

and we get the rule for mutually exclusive events

\[
P(A \text{ or } B) = P(A) + P(B)
\]

JOINT EVENTS

if we know \( P(A) \) and \( P(B) \)

what is \( P(A \text{ and } B) \)?

both events must occur (simultaneously or successively)

\[
P(3 \text{ on a die and HEAD on a coin flip})
\]

\[
\text{e.g., } P(3 \text{ or a 4 on a die}) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = \frac{1}{3}
\]

\[
1 2 3 4 5 6
\]
STATISTICAL INDEPENDENCE

Events are independent if the occurrence of one event does not affect the probability of the other event occurring.

E.g., rolling a 3 on a die has no effect on whether or not a coin will come up heads.

\[
P(3 \text{ on die}) = \frac{1}{6}
\]

\[
P(\text{HEADS}) = \frac{1}{2}
\]

So

\[
P(3 \text{ and HEADS}) = P(3) \times P(\text{HEADS})
\]

\[
P(3 \text{ and HEADS}) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}
\]

MULTIPLICATION

Why multiply probabilities of joint events?

Probability is the ratio of the number of outcomes including an event to the total number of possible outcomes.

For the joint event “3 on a die and HEADS”, the possible outcomes are 1H, 2H, 3H, 4H, 5H, 6H, 1T, 2T, 3T, 4T, 5T, 6T.

Count up the possibilities!

SAMPLING WITH REPLACEMENT

Suppose we have 10 numbered balls in a jar.

The probability of drawing ball 3 is \(\frac{1}{10}\).

If we put the ball back, the probability of drawing ball 3 again is \(\frac{1}{10}\) (same for any ball).

Each event (drawing ball 3) is independent from previous events.

In general for independent events \(A\) and \(B\),

\[
P(A \text{ and } B) = P(A) \times P(B)
\]

SAMPLING WITHOUT REPLACEMENT

Many times the probability of an event does depend on other events.

E.g., suppose we have ten numbered balls in a jar.

The probability of drawing ball 3 is \(\frac{1}{10}\).

Suppose we draw ball 2, leaving nine balls in the jar.

The probability of drawing ball 3 is now \(\frac{1}{9}\).

CONDITIONAL PROBABILITIES

We can describe the effect of other events by identifying conditional probabilities.

E.g.,

\[
P(\text{drawing ball 3 given that ball 2 was already drawn})
\]

\[
P(\text{ball 3|ball 2})
\]

In general, the probability of event \(A\), given event \(B\) is written as

\[
P(A|B)
\]

No direct way of calculating from \(P(A)\) or \(P(B)\).

NONINDEPENDENT EVENTS

When

\[
P(A) = P(A|B)
\]

we say events \(A\) and \(B\) are independent.

Otherwise the events are nonindependent (dependent).
JOINT PROBABILITY

if we know $P(A)$ and $P(B|A)$ then we can calculate the joint probability
$P(A \text{ and } B) = P(A)P(B|A)$

if we know $P(B)$ and $P(A|B)$ then we can calculate the joint probability
$P(A \text{ and } B) = P(B)P(A|B)$
same number!

if events are independent, this rule is the same as before because
$P(A|B) = P(A)$

EXAMPLE

what is the probability of drawing ball 2 and then ball 3 from a jar with ten numbered balls?
we know that
$P(\text{drawing ball 2 from the full jar}) = \frac{1}{10}$
$P(\text{drawing ball 3 | ball 2 is drawn from the full jar}) = \frac{1}{9}$
so
$P(\text{drawing ball 3 and drawing ball 2}) = P(\text{drawing ball 2 from the full jar}) \times P(\text{drawing ball 3 | ball 2 is drawn from the full jar}) = \frac{1}{10} \times \frac{1}{9} = \frac{1}{90}$

RANDOMNESS

we assume coin flips, rolling dice, samples from jars are random events
unpredictable for a specific instance
predictable on average over lots of samples (likelihood of happening)
randomness is sometimes a good thing

CONCLUSIONS

probability
mutually exclusive events
compound events
independence

NEXT TIME

review for exam
SECTION EXAM 1
fun problems with probability