INFERENTIAL STATISTICS

when we get a set of data it is either from all possible sources (population) or a subset of sources (sample)

with inferential statistics we take a random sample and try to infer something about the population

we want to do two things

1. test hypotheses about parameters (measures of the population).
2. estimate parameters.

2

SAMPLING

suppose we have a population with a mean $\mu$ and a standard deviation $\sigma$

suppose we take a sample from the population and calculate a sample mean $\overline{X}_1$

suppose we take a different sample from the population and calculate a sample mean $\overline{X}_2$

suppose we take a different sample from the population and calculate a sample mean $\overline{X}_3$

DISTRIBUTION

the different $\overline{X}_i$ sample means that are calculated will be related to each other because they all come from the same population, which has a population mean of $\mu$

we can consider a distribution of the sample means

(same idea as distribution of sum of dice roles)

2

DISTRIBUTION

this distribution involves frequencies of means rather than frequencies of scores

for most of inferential statistics we do not deal with the frequency distribution of scores

A sampling distribution is the distribution of values of the statistic under consideration, from all possible samples of a given size.

currently, the statistic is the sample mean $\overline{X}$
SAMPLING DISTRIBUTION

how do we get the sampling distribution?

e.g.
suppose you have a population of 5 people with math scores
and you take sample sizes of 3
you must consider every possible group of 3 people from the population
turns out there are 10 such groups

NOTE: the number of samples is greater than the size of the population!

CENTRAL LIMIT THEOREM

fortunately, there are theorems that tell us what the distribution will look like

as the sample size ($n$) increases, the sampling distribution of the mean for simple random samples of $n$ cases, taken from a population with a mean equal to $\mu$ and a finite variance equal to $\sigma^2$, approximates a normal distribution

another theorem based on unbiased estimation tells us that the mean of the sampling distribution is $\mu$

STANDARD ERROR

of course the standard deviation of the sampling distribution is the square root of the variance

$$\sigma_X = \sqrt{\frac{\sigma^2}{n}}$$
or

$$\sigma_X = \frac{\sigma}{\sqrt{n}}$$

also called the standard error of the mean

WHY BOTHER?

suppose you know that for a population, $\mu = 455$ and $\sigma = 100$

(an example involving SAT scores from your text)

then we know the following about a sampling distribution involving samples sizes of 144 students

1. The distribution is normal.
2. The mean of the distribution is 455.
3. The standard error of the mean is $100/\sqrt{144} = 8.33$.
PROBABILITY
we can answer questions like
what is the probability of randomly
selecting a sample with a mean $X$ such that
$440 < X < 460$?
area under the curve

PROBABILITY
everything is just like before
area under the curve
We use the normal distribution
calculator with Mean=455 and
SD=8.33

SAMPLING DISTRIBUTION
the sampling distribution has two
critical properties
1. As sample size ($n$) increases, the sam-
   pling distribution of the mean be-
   comes more like the normal distribu-
tion in shape, even when the popu-
lation distribution is not normal.
2. As the sample size ($n$) increases, the
   variability of the sampling distribu-
tion of the mean decreases (the stan-
der error decreases).

SHAPE
with large sample sizes, all sampling
distributions look like normal
distributions
means the conclusions we draw from
sampling distributions are not
dependent on the shape of the
population distribution!
a remarkable result that is due to the
central limit theorem

VARIABILITY
from our calculation of standard error:
$$\sigma_X = \frac{\sigma}{\sqrt{n}}$$
we see that increasing $n$ makes for
smaller values of $\sigma_X$
e.g. for $n = 144$ in our previous
example $\sigma_X = 8.33$

VARIABILITY
but if $n = 20$,
$$\sigma_X = \frac{100}{\sqrt{20}} = 22.36$$
compare to the 8.33 with $n = 144$
VARIABILITY

OR if \( n = 1000 \),
\[ \sigma_{\bar{X}} = \frac{100}{\sqrt{1000}} = 3.16 \]

compare to the 8.33 with \( n = 144 \)

increasing the sample size decreases
the variability of sample means
makes sense if you think about it

SAMPLING

to use the sampling distribution like
we want to, we must have random
samples

without random sampling, our
calculations about probability of
sample means are not valid
(this will get more important later)

lots of methods of sampling that
emphasize different aspects of the data

TYPES OF SAMPLING

more detail in PSY 203: Experimental Methods.

- simple random sampling
- systematic sampling
- cluster sampling
- stratified random sampling

WHY STATISTICS WORKS

we have two ways of finding the
sampling distribution of the mean

1. gather lots of samples, calculate means
   and standard deviations (virtually impossible)
2. calculate mean and standard deviation
   of the population, use central
   limit theorem (relatively easy)

the central limit theorem allows us to
do inferential statistics, without it,
much of this course would not exist
(actualy there is one other way to do statistics...)

CONCLUSIONS

sampling distribution looks like a
normal distribution

methods of calculating mean and
standard deviation if \( \mu \) and \( \sigma \) are known

samples must be randomly selected
NEXT TIME

using sampling distributions
evidence that the theorems work
and
Marvel at my predictive powers!