LAST TIME
we know how to check if a sample
mean, $X$, is statistically significantly
different from a hypothesized
population mean, $\mu$.
but sometimes we have no idea what $\mu$
is!
we would like to be able to estimate
$\mu$ using the sample data we have
statistical estimation

INTERVAL ESTIMATION
we get a better idea of the value of $\mu$
by considering a range of values that
are likely to contain $\mu$
we will show how to build up
confidence intervals using the
properties of the sampling distribution
of the mean

$\sigma$ KNOWN
to demonstrate our technique, suppose
we have a population of scores with
$\mu = 43$, $\sigma = 10$
from the population we get the
sampling distribution for samples of
size $n = 400$ with
$\sigma_X = \frac{\sigma}{\sqrt{n}} = 0.50$

PORTFOLIO ESTIMATION
single value that represents the best
estimate of a population value
when we want to estimate $\mu$, the best
point estimate is the sample mean $\bar{X}$
but the estimate depends on which
sample we select!
CONFIDENCE INTERVALS

construct an interval around the observed statistic, $\bar{X}$

$CI = \text{statistic} \pm (\text{critical value}) (\text{standard error of the statistic})$

$CI = \bar{X} \pm (t_{cv}) (s_{\bar{X}})$

- $\bar{X}$ is the sample mean
- $t_{cv}$ is the critical value using the appropriate $t$ distribution for the desired level of confidence
- $s_{\bar{X}}$ is the estimated standard error of the mean

$\hat{s}_{\bar{X}} = \frac{s}{\sqrt{n}}$

LEVEL OF CONFIDENCE

degree of confidence that computed interval contains $\mu$

usually complement of level of significance, $\alpha$

level of confidence is $(1 - \alpha)$

calculating the critical value $t_{cv}$ is the same!

e.g., for $\alpha = 0.05$, $(1 - \alpha) = 0.95$, and $t_{cv} = 1.9659$

(using the Inverse $t$ Distribution calculator with df=399)

CONFIDENCE INTERVAL

suppose we calculate $\bar{X} = 44.6$

the confidence interval is then

$CI = \bar{X} \pm (t_{cv})(s_{\bar{X}})$

$CI_{.95} = 44.6 \pm (1.9659)(0.50)$

$CI_{.95} = (43.62, 45.58)$

CONFIDENCE INTERVAL

this means we are 95% confident that the interval $(43.62, 45.58)$ contains the unknown value $\mu$

note: if $\mu = 43$ like was said originally, we are wrong!

CI does not contain $\mu$ (no way to avoid error completely)!

EXAMPLE

Guess the height of this room in feet, and write down your guess on a piece of paper

Now go around the room and get 10 guesses from other random people

Then, tell me your guess

Calculate the mean and standard deviation for your sample

$\bar{X} = \frac{\sum X_i}{n}$

$s = \sqrt{\frac{\sum X_i^2 - \left(\sum X_i\right)^2/n}{n-1}}$

I’ll calculate the population mean for the class each of you will calculate a confidence interval, for your sample, with $\alpha = 0.05$

so from the Inverse $t$ Distribution Calculator, we find that $t_{cv} = 2.262$
CONFIDENCE INTERVALS

thus

\( \text{CI}_{95} = \bar{X} \pm (t_{\alpha})(s_{\bar{X}}) \)

\( \text{CI}_{95} = \bar{X} \pm (2.262)(s_{\bar{X}}) \)

\( \text{CI}_{95} = ( , ) \)

WHAT DOES THIS MEAN?

we conclude with 95% confidence that your interval contains \( \mu \)

this is a probabilistic statement about the interval

\( \mu \) is a parameter, a fixed number

different samples produce different confidence intervals, but 95% of the time the interval would contain \( \mu \)

check

CONFIDENCE

we never say that a specific confidence interval contains \( \mu \) with probability 0.95

either the interval contains \( \mu \) or it does not

we can say that the procedure of producing CI’s produce intervals that contain \( \mu \) with probability 0.95

we do talk about the confidence that an interval includes \( \mu \)

we would say that the confidence interval contains \( \mu \) with confidence of 0.95

the confidence is in the procedure of calculating CIs

CONCLUSIONS

estimation

confidence intervals

normal distribution, \( t \) distribution

interpretation

NEXT TIME

more on estimation

relationship between confidence intervals and hypothesis testing

statistical precision

Less than 5% of published psychological research should be wrong (and why that probably isn’t true).