PSY 201: Statistics in Psychology

Lecture 15
Signal detection
Making decisions.

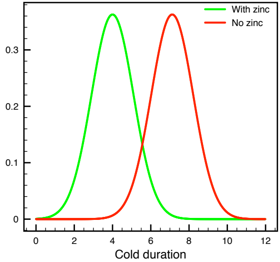
Greg Francis

Purdue University

Fall 2023

ZINC AND COLDS

- Distributions of cold duration when taking zinc or not taking zinc overlap somewhat



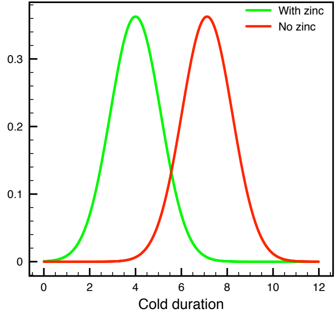
$$d' = \frac{\mu_{NZ} - \mu_{WZ}}{\sigma} = \frac{7.12 - 4.00}{1.1} = 2.02$$

ZINC AND COLDS

- Suppose you sample a person who has a cold and find the duration.
- Using just that information, you want to decide whether the person took zinc or not (e.g., you advised your friend to take the zinc, but he bought a generic version on the Internet and you suspect the tablets do not actually contain zinc).
    - If the cold duration is long, you conclude the tablets do not contain zinc
    - If the cold duration is short, you conclude the tablets do contain zinc

ZINC AND COLDS

- Distributions of cold duration when taking zinc or not taking zinc overlap somewhat



- We want to define a *decision criterion* to separate short and long cold durations
- Suppose we set our criterion to be

$$C = 4$$

DECISION OUTCOMES

	State of nature	
Decision made	Tablets contain zinc	Tablets do not contain zinc
Decide tablets contain zinc	Hit	False Alarm
Decide tablets do not contain zinc	Miss	Correct Rejection

- When making decisions in noise there is **always** the risk of making errors!
- We want to think about the probability of different outcomes

WITH ZINC

- Suppose the tablets really do contain zinc, then when you make a decision you either make:
  - Hit (if you decide the tablets contain zinc)
  - Miss (if you decide the tablets do not contain zinc)
- We know $\mu_{WZ} = 4$ and $\sigma = 1.1$. If we use a criterion of $C = 4$, how often do we make hits and misses?
- (Use the on-line calculator)
  - Hit: P(decide contains zinc — tablet contains zinc) $= 0.5$
  - Miss: P(decide no zinc — tablet contains zinc) $= 0.5$

NO ZINC

- Suppose the tablets really do not contain zinc, then when you make a decision you either make:
  - False Alarm (if you decide the tablets contain zinc)
  - Correct Rejection (if you decide the tablets do not contain zinc)
- We know $\mu_{NZ} = 7.12$ and $\sigma = 1.1$. If we use a criterion of $C = 4$, how often do we make false alarms and correct rejections?
- (Use the on-line calculator)
  - False Alarm: P(decide contains zinc — tablet has no zinc) $= 0.0023$
  - Correct Rejection: P(decide no zinc — tablet has no zinc) $= 0.9977$

DECISION OUTCOMES

$$P(\text{correct decision}) = P(\text{decide contains zinc}|\text{tablet contains zinc}) \times P(\text{tablet contains zinc}) +$$
$$P(\text{decide no zinc}|\text{tablet has no zinc}) \times P(\text{tablet has no zinc})$$

- If it is equally likely that the tablets contain zinc or do not contain zinc, then the probability that you make a correct decision is:

$$0.5 \times 0.5 + 0.9977 \times 0.5 = 0.74885$$

DIFFERENT CRITERION

- Suppose the tablets really do contain zinc; we know $\mu_{WZ} = 4$ and $\sigma = 1.1$. If we use a criterion of $C = 5$, how often do we make hits and misses?
  - Hit: $P(\text{decide contains zinc}|\text{tablet contains zinc}) = 0.8183$
  - Miss: $P(\text{decide no zinc}|\text{tablet contains zinc}) = 0.1817$
- Suppose the tablets really do not contain zinc; we know $\mu_{NZ} = 7.12$ and $\sigma = 1.1$. If we use a criterion of $C = 5$, how often do we make false alarms and correct rejections?
  - False Alarm: P(decide contains zinc — tablet has no zinc) $= 0.027$
  - Correct Rejection: P(decide no zinc — tablet has no zinc) $= 0.973$

DECISION OUTCOMES

$$P(\text{correct decision}) = P(\text{decide contains zinc}|\text{tablet contains zinc}) \times P(\text{tablet contains zinc}) +$$

$$P(\text{decide no zinc}|\text{tablet has no zinc}) \times P(\text{tablet has no zinc})$$

- If it is equally likely that the tablets contain zinc or do not contain zinc, then the probability that you make a correct decision is:

$$0.8183 \times 0.5 + 0.973 \times 0.5 = 0.89565$$

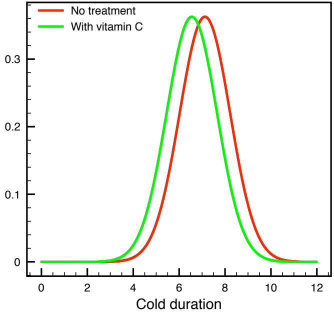
- Using $C = 5$ produces better outcomes (more likely to make the right decision) than using $C = 4$.
- What would be the **optimal** criterion?

TRADE OFFS

- Setting the decision criterion always involves trade offs. In our situation of cold durations and zinc in tablets:
  - Increasing $C \rightarrow$ more hits, more false alarms
  - Deceasing $C \rightarrow$ more misses, more correct rejections
- You generally cannot avoid some errors when making decisions under noisy situations

OVERLAP

- For vitamin C, the durations overlap quite a bit



- We take the mean of the "no treatment" distribution (noise alone) and compute distance to the mean of the "with vitamin C" distribution
- in standardized units

$$d' = \frac{\mu_{NT} - \mu_{WC}}{\sigma} = \frac{7.12 - 6.55}{1.1} = 0.52$$

OVERLAP

- Suppose the tablets really do contain vitamin C; we know $\mu_{WC} = 6.55$ and $\sigma = 1.1$. If we use a criterion of $C = 5$, how often do we make hits and misses?
  - Hit: P(decide contains vitamin C|tablet contains vitamin C) $= 0.0794$
  - Miss: P(decide no vitamin C|tablet contains vitamin C) $= 0.9206$
- Suppose the tablets really do not contain vitamin C; we know $\mu_{NT} = 7.12$ and $\sigma = 1.1$. If we use a criterion of $C = 5$, how often do we make false alarms and correct rejections?
  - False Alarm: P(decide contains vitamin C|tablet has no vitamin C) $=$ 0.027
  - Correct Rejection: P(decide no vitamin C|tablet has no vitamin C) $=$ 0.973

DECISION OUTCOMES

$$P(\text{correct decision}) =$$

$$P(\text{decide contains vitamin C}|\text{tablet contains vitamin C}) \times P(\text{tablet contains vitamin C}) +$$

$$P(\text{decide no vitamin C}|\text{tablet has no vitamin C}) \times P(\text{tablet has no vitamin C})$$

- If it is equally likely that the tablets contain vitamin C or do not contain vitamin C, then the probability that you make a correct decision is:

$$0.0794 \times 0.5 + 0.973 \times 0.5 = 0.5262$$

- Not much better than a random guess!

OVERLAP

- Suppose the tablets really do contain vitamin C; we know $\mu_{WC} = 6.55$ and $\sigma = 1.1$. If we use a criterion of $C = 6.835$ (optimal), how often do we make hits and misses?
  - Hit: P(decide contains vitamin C|tablet contains vitamin C) = 0.6022
  - Miss: P(decide no vitamin C|tablet contains vitamin C) = 0.39778
- Suppose the tablets really do not contain vitamin C; we know $\mu_{NT} = 7.12$ and $\sigma = 1.1$. If we use a criterion of $C = 6.835$, how often do we make false alarms and correct rejections?
  - False Alarm: P(decide contains vitamin C|tablet has no vitamin C)= 0.39778
  - Correct Rejection: P(decide no vitamin C|tablet has no vitamin C) = 0.6022

DECISION OUTCOMES

$$P(\text{correct decision}) =$$

$$P(\text{decide contains vitamin C}|\text{tablet contains vitamin C}) \times P(\text{tablet contains vitamin C}) +$$

$$P(\text{decide no vitamin C}|\text{tablet has no vitamin C}) \times P(\text{tablet has no vitamin C})$$

- If it is equally likely that the tablets contain vitamin C or do not contain vitamin C, then the probability that you make a correct decision is:

$$0.6022 \times 0.5 + 0.6022 \times 0.5 = 0.6022$$

- Not great, but you cannot do better!

CONCLUSIONS

- signal-to-noise ratio
- decision criterion
- decision outcomes
- performance
- trade-offs

NEXT TIME

- Underlying distributions

Can you read my mind?