PSY 201: Statistics in Psychology

Lecture 24

Hypothesis testing for correlations

Is there a correlation between homework and exam grades?

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HYPOTHESIS TESTING

- four steps
  1. State the hypothesis and the criterion
  2. Compute the test statistic.
  3. Compute the p-value.
  4. Make a decision

HYPOTHESIS TESTING

- we need to know the properties of the sampling distribution
- for the mean, the central limit theorem tells us that the sampling distribution is normal, and specifies the mean and standard deviation (standard error)
- area under the curve of the sampling distribution gives probability of getting that sampled value, or values more extreme (*p*-value)
- for other types of statistics, the sampling distribution is different
  - area under the curve of sampling distribution still gives probability of getting that sampled value, or values more extreme
- correlation coefficient

HYPOTHESIS TESTING

- the approach is still basically the same
- we compute

$$\text{Test statistic} = \frac{\text{statistic - parameter}}{\text{standard error of the statistic}}$$

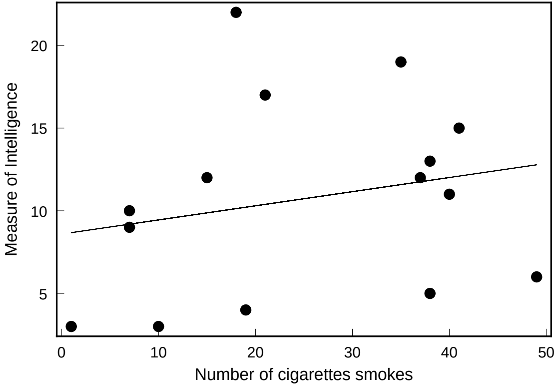
- and use it to compute a *p*-value, which we compare to α

CORRELATION COEFFICIENT

- from a population with scores X and Y, we can calculate a correlation coefficient
- let ρ be the correlation coefficient **parameter** of the population
- let r be the correlation coefficient **statistic** from a random sample of the population

SAMPLING

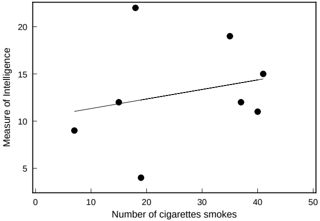
- Suppose $\rho = 0.22$



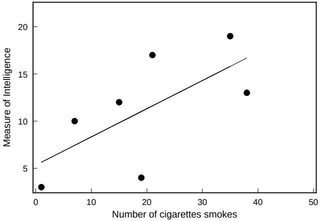
- depending on which points we sample, the computed r will take different values

RANDOM SAMPLING

- $r = 0.24$

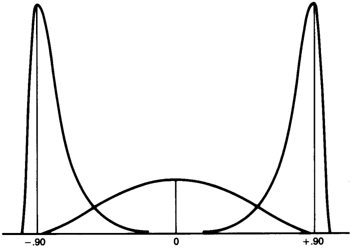


- $r = 0.67$



SAMPLING DISTRIBUTION

- frequency of different r values, given a population parameter ρ
- **not** usually a normal distribution!
- often skewed to the left or the right
- cannot find area under curve!

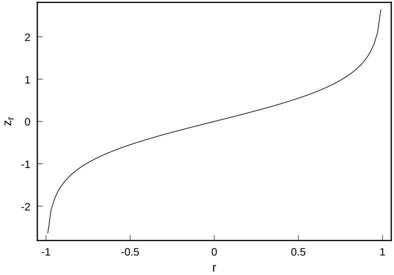


FISHER z TRANSFORM

- formula for creating new statistic

$$z_r = \frac{1}{2} \log_e \left(\frac{1+r}{1-r} \right)$$

- where $\log_e$ is the "natural logarithm" function
  - also sometimes designated as ln



- textbook provides a r to z' calculator (reversible!)

FISHER z TRANSFORM

- for large samples, the sampling distribution of z_r is normally distributed
  - regardless of the value of ρ
- with a mean

$$z_\rho = \frac{1}{2}\log_e\left(\frac{1+\rho}{1-\rho}\right)$$

- and with standard error (standard deviation of the sampling distribution)

$$s_{z_r} = \sqrt{\frac{1}{n-3}}$$

- where n is the sample size

FISHER z TRANSFORM

- means we can use all our knowledge about hypothesis testing with normal distributions for the transformed scores!
- online calculator converts r to z_r (it calls it z')
- e.g.

$$r = -0.90 \rightarrow z_r = -1.472$$

$$r = 0 \rightarrow z_r = 0$$

$$r = 0.45 \rightarrow z_r = 0.485$$

- we can convert back the other way from $z_r \rightarrow r$ too!

HYPOTHESIS TESTING

- Suppose we study a population of data that we think has a correlation of 0.65. We want to test the hypothesis with a sample size of $n = 30$.
- e.g. family income and attitudes about democratic childrearing
- Step 1. State the hypothesis and criterion

$$H_0 : \rho = 0.65$$

$$H_a : \rho \neq 0.65$$

- two-tailed test

$$\alpha = 0.05$$

HYPOTHESIS TESTING

- Step 2. Compute the test statistics
- suppose from our sampled data we get

$$r = 0.61$$

- we need to convert it to a z_r score

$$r = 0.61 \rightarrow z_r = 0.709$$

- and calculate standard error

$$s_{z_r} = \sqrt{\frac{1}{n-3}} = \sqrt{\frac{1}{27}} = 0.192$$

HYPOTHESIS TESTING

- now we calculate the test statistic

$$\text{Test statistic} = \frac{\text{statistic - parameter}}{\text{standard error of the statistic}}$$

$$z = \frac{z_r - z_\rho}{s_{z_r}} = \frac{0.709 - 0.775}{0.192} = -0.344$$

- Step 3. Compute the p-value. From the Normal Distribution calculator, we compute

$$p = 0.7346$$

HYPOTHESIS TESTING

- Step 4. Make a decision.

$$p = 0.7346 > 0.05 = \alpha$$

- H_0 is **not** rejected at the 0.05 significance level
  - The probability of getting $r = 0.61$ (or a value further away from 0) with a random sample, if $\rho = 0.65$, is greater than 0.05.
  - The observed difference is not a significant difference.

A SPECIAL CASE

- hypothesis testing of correlation coefficients can always use Fisher's z transform

$$H_0 : \rho = a$$

$$H_a : \rho \neq a$$

- special case $a = 0$:

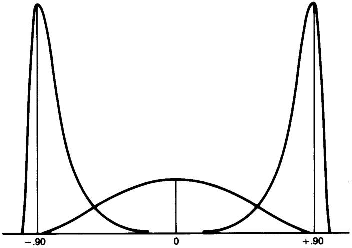
$$H_0 : \rho = 0$$

$$H_a : \rho \neq 0$$

- Is there a significant correlation coefficient?
- Is there a linear relationship between two variables?

SAMPLING DISTRIBUTION

- while we needed Fisher's z transformation to convert the sampling distribution into a normal distribution
- it is not necessary for testing $\rho = 0$



SAMPLING DISTRIBUTION

- for $\rho = 0$ the sampling distribution of the test statistic is a t distribution with df $= n - 2$
  - two sets of scores, minus 1 from each set
- no need to convert with Fisher z transform
- we follow the same procedures as before
  1. State the hypothesis. $H_0 : \rho = 0$ and set the criterion
  2. Compute the test statistic.
  3. Compute the p-value
  4. Make a decision.

HYPOTHESIS TESTING

- everything is the same, except the test statistic calculation is a bit different
- it turns out that an estimate of the standard error is:

$$s_r = \sqrt{\frac{1 - r^2}{n - 2}}$$

- so that the test statistic is:

$$t = \frac{r - \rho}{s_r} = r\sqrt{\frac{n - 2}{1 - r^2}}$$

- we use this with a t distribution to compute a p-value

EXAMPLE

- $n = 32$ scores calculated to get $r = -0.375$

1. State the hypothesis. $H_0 : \rho = 0$, $H_a : \rho \neq 0$, $\alpha = 0.05$
2. Compute the test statistic

$$t = r\sqrt{\frac{n-2}{1-r^2}} = (-0.375)\sqrt{\frac{30}{0.859}} = -2.216$$

3. Compute the p value using the t Distribution calculator with df$=n-2=30$

$$p = 0.0344$$

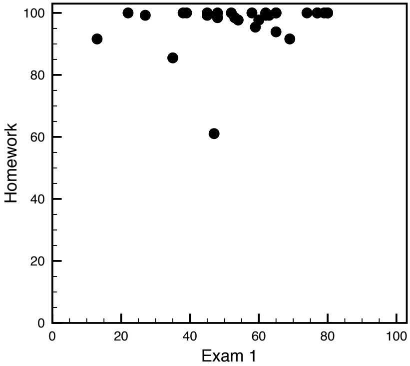
4. Interpret the results: $p = 0.0344 < 0.05 = \alpha$; reject H_0

EXAMPLE

- I took the percentage of the first five homework grades and correlated it with the first exam scores

$$\rho = 0.2123$$

- Is this a significant correlation?



CAREFULL!

- If I treat the class as a *population*, the correlation simply is what it is. Significance is not an issue!
- If I treat the class as a *sample* of students who do homework and take exams in statistics, then I can ask about statistical significance

CAREFULL!

- is $r = 0.2123$ significantly different from 0? I have $n = 30$ scores
- Compute the test statistics.

$$t = r\sqrt{\frac{n-2}{1-r^2}} = 1.1496$$

- use the t Distribution calculator with df$=n-2=28$

$$p = 0.769$$

- Interpret the results: $p = 0.26 > 0.05 = \alpha$, do not reject H_0

READING?

- For Homework and Reading, $r = 0.8964$. I have $n = 30$ scores
- Compute the test statistics.

$$t = r\sqrt{\frac{n-2}{1-r^2}} = 10.70$$

- use the t Distribution calculator with df$=n-2=28$

$$p \approx 0$$

- Interpret the results: $p \approx 0 < 0.05 = \alpha$, reject H_0

CAREFUL!

- When we conclude a test is statistically significant, we base that on the observation that observed data (or more extreme) would be rare if the H_0 were true
- But if we make multiple tests from a single sample, our calculations of probability may be invalid.
- We performed two hypothesis tests from one sample of students.
- Each test has a chance of producing a significant results, even if H_0 is true
- It is not appropriate to just run various tests with one data set, if all you are doing is looking for significant results (fishing)
- You have to do a different type of statistical analysis

CONFIDENCE INTERVAL

- Always use the Fisher z transform
- Build interval as a Fisher z score and then convert to correlation (r value)

$$CI = z_r \pm z_{cv} s_{z_r}$$

- the correlation between homework and reading scores:

$$CI_{95} = 1.453 \pm (1.96)(0.192) = (1.076, 1.831)$$

- when we convert to r values:

$$(0.792, 0.950)$$

POWER

- How would we design a good experiment to test a correlation?
- How big a sample do we need to have a 90% chance of rejecting the H_0?
- Conceptually, this is the same issue as estimating power or sample size for a hypothesis test of means
- We just need to use the sampling distribution for the Fisher z transform of the sample correlation instead of the sampling distribution for a sample mean

POWER

- We have to specify the specific correlation for the alternative hypothesis
- Suppose we plan to test

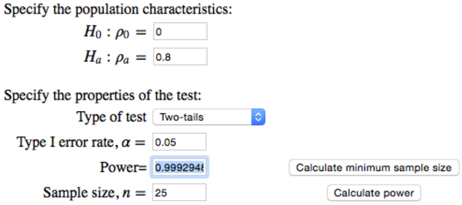
$$H_0 : \rho = 0, H_a : \rho \neq 0$$

- and we set the specific alternative as

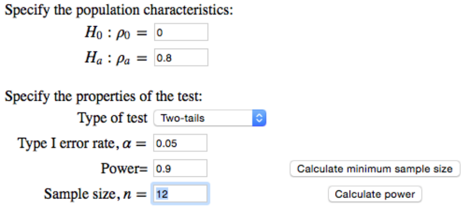
$$H_a : \rho_a = 0.8$$

- What is the probability that a random sample of $n = 25$ will reject the H_0?
- The on-line calculator does all the work!

POWER



- Higher than 99.9% chance of rejecting the null hypothesis
- What sample size do we need to have 90% power?



- However, whether these calculations make sense depends on whether $\rho = 0.8$ in reality.

CONCLUSIONS

- correlation coefficient
- Fisher z transform
- testing significance of correlation
- confidence interval
- power

NEXT TIME

- hypothesis testing of two means
- homogeneity of variance
- confidence interval
- robustness and assumptions

Check yourself before you wreck yourself.