Exploration of Contextuality in a Psychophysical Double-Detection Experiment

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Abstract. The Contextuality-by-Default (CbD) theory allows one to separate contextuality from context-dependent errors and violations of selective influences (aka "no-signaling" or "no-disturbance" principles). This makes the theory especially applicable to behavioral systems, where violations of selective influences are ubiquitous. For cyclic systems with binary random variables, CbD provides necessary and sufficient conditions for noncontextuality, and these conditions are known to be breached in certain quantum systems. We apply the theory of cyclic systems to a psychophysical double-detection experiment, in which observers were asked to determine presence or absence of a signal property in each of two simultaneously presented stimuli. The results, as in all other behavioral and social systems previous analyzed, indicate lack of contextuality. The role of context in double-detection is confined to lack of selectiveness: the distribution of responses to one of the stimuli is influenced by the state of the other stimulus.

Keywords: contextuality, cyclic systems, inconsistent connectedness, psychophysics.

The Contextuality-by-Default (CbD) theory [9,10] describes systems of measurements with respect to the conditions under which they are recorded and determines the tenability of a non-contextual description of the system. In this paper, we study the double-detection paradigm suggested in Refs. [6] and [8]. In this paradigm, two stimuli are presented to an observer simultaneously (left-right), each on one of several possible levels. The observer is asked to state (Yes/No), for each of the two observation areas, whether it contains a particular target property (signal). The signal is objectively present in a subset of levels of a stimulus. When such experimental situation includes only two levels for each stimulus (e.g., present/absent), the system of measurements is formally equivalent to that of the Einstein–Podolski–Rosen/Bohm (EPR/B) paradigm (see e.g, Ref. [6]).

1 Contextuality in CbD

We briefly recapitulate the concepts of the CbD, to make this paper self-sufficient. For detailed discussions see Ref. [9] and Ref. [10]; the proofs may be found in Refs. [11, 14, 15].

Definition 1. (System of measurements) *A* system of measurements *is a matrix $\mathfrak{R}_{n\times m}$, in which columns correspond to the properties $\{q_1, \dots, q_n\}$ and rows to the contexts $\{c_1, \dots, c_m\}$. A cell (i, j) contains the random variable R_i^j if q_i is measured in context c_j, and the cell is left empty otherwise.*

When adopting the CbD framework, the first goal is to produce a matrix $\mathfrak{R}$ that formally represents the experiment and its results.

Definition 2. (Connections and bunches) *The random variables in any column of a system of measurements form a* connection *for the corresponding property; denote the connection for property q_i by $\mathfrak{R}_i$. Those in any row form a* bunch *representing the corresponding context; denote the bunch for context c_j by R^j.*

Note that elements of a connection are necessarily ("by default") pairwise distinct and pairwise stochastically unrelated, i.e., no R_i^j and R_i^k with $k \neq j$ have a joint distributions. Consequently, the system $\mathfrak{R}$ does not have a joint probability distribution including all of its elements. See Refs. [5, 10].

Definition 3. (Coupling) *Let X_i, with $i \in \mathrm{I}$, an index set, be a random variable on a probability space $(\mathrm{X}_i, \Sigma_i, P_i)$. Let $\{Y_i : i \in \mathrm{I}\}$ be a collection of jointly distributed random variables (i.e., a random variable in its own right) on a probability space (Y, Ω, p). The random variable $\{Y_i : i \in \mathrm{I}\}$ is called a* coupling *of the collection $\{X_i : i \in \mathrm{I}\}$ if for all $i \in \mathrm{I}$, $Y_i \overset{d}{=} X_i$, where $\overset{d}{=}$ denotes identity in distribution.*

Definition 4. (Maximal coupling) *Let $Y = (Y_i : i \in \mathrm{I})$ be a coupling of a collection $\{X_i : i \in \mathrm{I}\}$. And let M be the event where $\{Y_i = Y_j$ for all $i, j \in \mathrm{I}\}$. If $\Pr(\mathrm{M})$ is the largest possible among all couplings of $\{X_i : i \in \mathrm{I}\}$, then Y is a* maximal coupling *of $\{X_i : i \in \mathrm{I}\}$.*

Definition 5. (Contextual system) *Let $\mathfrak{R}$ be a system of measurements. Let S be a coupling of $\mathfrak{R}$ such that for each $c_j \in \{c_1, \dots, c_m\}$, S^j is a coupling of R^j contained in S. The system $\mathfrak{R}$ is said to be* non-contextual *if it has a coupling S such that for all $q_i \in \{q_1, \dots, q_n\}$, the coupling S_i is a maximal coupling.*

Definition 6. (Cyclic system with binary variables) *Let $\mathfrak{R}$ be a system of measurements such that (a) each context contains two properties; (b) each property is measured in two different contexts; (c) no two contexts share more than one property; and (d) each measurement is a binary random variable, with values ± 1. Then the system $\mathfrak{R}$ is a* cyclic system with binary variables *and in the following will be simply called* a cyclic system.

Remark 1. Note that a cyclic system is composed of the same number n of connections and of bunches, and it contains $2n$ random variables. We shall say that a cyclic system has rank n or is of rank n to explicitly refer to this number.

Definition 7. (Consistent connections) *Let $\mathfrak{R}_i$ be a connection in a system $\mathfrak{R}$. It is said that $\mathfrak{R}_i$ is a* consistent connection *if for all $c_j, c_k \in \{c_1, \ldots, c_m\}$ such that R_i^j and R_i^k are defined (i.e., both cells (i,j) and (i,k) of $\mathfrak{R}$ are not empty), $R_i^j \overset{d}{=} R_i^k$.*

Definition 8. (Consistently connected system) *A system of measurements $\mathfrak{R}$ is said to be* consistently connected *if for all $q_i \in \{q_1, \ldots, q_n\}$, the connection $\mathfrak{R}_i$ is a consistent connection. For a cyclic system, define*

$$ICC = \sum_{i=1}^{n} \left| \left\langle R_i^j \right\rangle - \left\langle R_i^k \right\rangle \right| .$$

ICC provides a measure of how inconsistent the connections are in the system.

Definition 9. (Contextuality in cyclic systems) *Let $\mathfrak{R}$ be a cyclic system with n binary variables. Let*

$$\mathsf{s}_1(x_1, x_2, \ldots, x_n) = \max \left\{ \sum_{k=1}^{n} a_k x_k : a_k = \pm 1 \text{ and } \prod_{k=1}^{n} a_k = -1 \right\} .$$

Let

$$\Lambda C = \mathsf{s}_1 \left(\left\{ \left\langle R_i^j R_{i'}^j \right\rangle : q_i, q_{i'} \text{ measured in } c_j, \text{ and } c_j \in \{c_1, \ldots, c_m\} \right\} \right)$$

Let $\Delta C = \Lambda C - ICC - (n-2)$. The quantity ΔC is a measure of contextuality *for cyclic systems.*

Theorem 1. (Cyclic system contextuality criterion, [14]) *A cyclic system $\mathfrak{R}$ is contextual if and only if $\Delta C > 0$.*

Remark 2. ΔC for a consistently connected cyclic system with $n = 4$ reduces to the Bell/CHSH inequalities [3, 10].

2 Contextuality in Behavioral and Social Data

In Ref. [13] many empirical studies of behavioral and social systems were reviewed. Most of those systems come from social data; that is, an observation for each measurement was the result of posing a question to a person, and the set of observations comes from questioning groups of people. For all the studies considered there, the CbD analyses showed that the systems, treated as cyclic systems ranging from rank 2 to 4, were non-contextual. Only one of the studies reviewed in Ref. [13] dealt with responses from a single person to multiple replications of stimuli.

Now, a key modeling problem in cognitive psychology has been determining whether a set of inputs selectively influences a set of response variables (Refs. [4, 16–18]). The formal theory of selective influences has been developed for the case of consistent connectedness, which has been treated as a necessary condition of selective influences; it follows from this formalism that selectiveness of influences in a consistently connected system is negated precisely in the case where it is contextual [6].

However, in most, if not all, behavioral systems, some form of influence upon a given random output is expected from most, if not all, of the system's inputs (Ref. [17]). This means that in the behavioral domain inconsistently connected systems are ubiquitous. While the presence of inconsistent connections rules out the possibility of selective influences, it does not imply that the full behavior of the system is accounted for by the direct action of inputs upon the outputs; an inconsistently connected behavioral system may still be contextual in the sense of CbD.

The double detection paradigm suggested in [6] and [8] provides a framework where both (in)consistent connectedness and contextuality can be studied in a manner very similar to how they are studied in quantum-mechanical systems (or could be studied, because consistent connectedness in quantum physics is often assumed rather than documented).

3 Method

3.1 Participants

Three volunteers, two females and one male, graduate students at Purdue University, served as participants for the experiment, including the first author of this paper. They were recruited and compensated in accordance to Purdue University's IRB protocol #1202011876, for the research study "Selective Probabilistic Causality As Interdisciplinary Methodology" under which this experiment was conducted. All participants reported normal or corrected to normal vision and were aged around 30. They are identified as $P1 - P3$ in the text and their experience with psychophysical experiments ranged from none to more than three previous participations.

3.2 Apparatus

The experiment was run using a personal computer with an Intel® Core™ processor running Windows XP, a 24-in. monitor with a resolution of 1920×1200 pixels (px), and a standard US 104-key keyboard. A chin-rest with forehead support was used so that the distance between subject and monitor was kept at 90 cm; this made each pixel on the screen to occupy about 62 sec arc of the subjects' visual field.

3.3 Stimuli

The stimuli were similar to those from Refs. [1] and [12]. They consisted of two circles drawn in solid grey (RGB 100, 100, 100) on a black background in a computer screen, with a dot drawn at or near their center. The circles radius was 135 px with their centers 320 px apart; the dots and circumference lines were 4 px wide. The offset of each dot with respect to the center of each circle, when they were not presented at the center, was 4 px. An example of the stimuli (in reversed contrast) is shown in Figure 1.

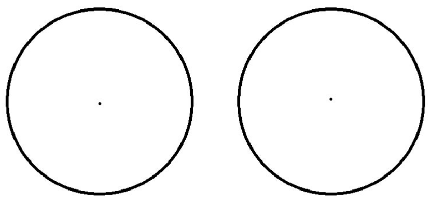

Fig. 1. Stimulus example

3.4 Procedure

Each participant performed nine experimental sessions. At the beginning of each experimental session, the chin-rest and chair heights were adjusted so that the subject could sit and use the keyboard comfortably. The time available for each session was 30 minutes, during which the participants responded in 560 (non-practice) trials (except for participant $P3$ in the sixth session, who only responded in 557 trials) preceded by up to 30 practice trials. The number of practice trials was set to 30 during the first two sessions and reduced to 15 during subsequent sessions. After each practice trial, the subject received feedback about whether their response for each circle was correct or not. The responses to practice trials were excluded from the analyses. Additionally, depending on their previous experience in psychophysical experiments the participants had up to three training sessions, also excluded from subsequent analyses.

Instructions for the experiment were presented to each participant verbally and written in the screen. In each trial the participant was required to judge for each circle whether the dot presented was displaced from the center or not. The stimuli were displayed until the subject produced their response. The responses were given by pressing and holding together two keys, one for each circle. Then, the dots in each circle were removed and a "Press the space bar to continue" message was flashed on top of the screen. After pressing the space bar, the message was removed and the next stimuli pair were presented after 400 ms. (Reaction times were measured from the onset of stimulus display until a valid response was recorded, but they were not used in the data analysis.)

3.5 Experimental Conditions

In each of two circles the dot presented could be located either at its center, or 4 px above, or else 4 px under the center. These locations produce a total of nine experimental conditions.

During each session, excepting the practice trials, the dot was presented at the center in a half of the trials; above the center in a quarter of them; and below the center in the remaining quarter, for each of the circles. Table 1 presents the proportions of allocations of trials to each of the 9 conditions.

Table 1. Probabilities with which a trial was allocated to one of the 9 experimental conditions.

	Center	Up	Down
Center	1/4	1/8	1/8
Up	1/8	1/16	1/16
Down	1/8	1/16	1/16

For each session, each trial was randomly assigned to one of the conditions in accordance with Table 1. The number of experimental sessions was chosen so that the expected number of (non-practice) trials in the conditions with lowest probabilities was at least 300. This number of observations was chosen based on Refs. [2], whose results show that coverage errors with respect to nominal values are below 1% for almost all confidence intervals for proportions with $n > 300$.

4 Analyses

Based on the experimental design depicted in Table 1, we specify the following properties:

- l_c: a dot is presented in the center of the left circle;
- r_c: a dot is presented in the center of the right circle;
- l_u: a dot is presented above the center of the left circle;
- r_u: a dot is presented above the center of the right circle;
- l_d: a dot is presented below the center of the left circle; and
- r_d: a dot is presented below the center of the right circle.

The 9 experimental conditions (contexts) then are denoted $l_c r_c, l_c r_u$, etc. Thus, the system of measurements depicted by the matrix in Figure 2 represents the complete 3×3 design given in Table 1.

$$
\begin{array}{ccccccc}
 & l_c & r_c & l_u & r_u & l_d & r_d \\
l_c r_c & R_{l_c}^{l_c r_c} & R_{r_c}^{l_c r_c} & \cdot & \cdot & \cdot & \cdot \\
l_u r_c & \cdot & R_{r_c}^{l_u r_c} & R_{l_u}^{l_u r_c} & \cdot & \cdot & \cdot \\
l_u r_u & \cdot & \cdot & R_{l_u}^{l_u r_u} & R_{r_u}^{l_u r_u} & \cdot & \cdot \\
l_d r_u & \cdot & \cdot & \cdot & R_{r_u}^{l_d r_u} & R_{l_d}^{l_d r_u} & \cdot \\
l_d r_d & \cdot & \cdot & \cdot & \cdot & R_{l_d}^{l_d r_d} & R_{r_d}^{l_d r_d} \\
l_c r_u & R_{l_c}^{l_c r_u} & \cdot & \cdot & R_{r_u}^{l_c r_u} & \cdot & \cdot \\
l_u r_d & \cdot & \cdot & R_{l_u}^{l_u r_d} & \cdot & \cdot & R_{r_d}^{l_u r_d} \\
l_d r_c & \cdot & R_{r_c}^{l_d r_c} & \cdot & \cdot & R_{l_d}^{l_d r_c} & \cdot \\
l_c r_d & R_{l_c}^{l_c r_d} & \cdot & \cdot & \cdot & \cdot & R_{r_d}^{l_c r_d}
\end{array}
$$

Fig. 2. System of measurements for double detection experiment.

We approach the exploration of this system through the theory of contextuality for cyclic systems in two ways. Firstly, note that from the system in Figure 2 we can extract six different cyclic subsystems of rank 6 and nine of rank 4. One of the rank 4 subsystems is presented in the left matrix in Figure 3. One of the rank 6 subsystems is shown in the right matrix in Figure 3.

$$
\begin{array}{ccccc}
 & l_c & r_c & l_u & r_u \\
l_c r_c & R_{l_c}^{l_c r_c} & R_{r_c}^{l_c r_c} & \cdot & \cdot \\
l_u r_c & \cdot & R_{r_c}^{l_u r_c} & R_{l_u}^{l_u r_c} & \cdot \\
l_u r_u & \cdot & \cdot & R_{l_u}^{l_u r_u} & R_{r_u}^{l_u r_u} \\
l_c r_u & R_{l_c}^{l_c r_u} & \cdot & \cdot & R_{r_u}^{l_c r_u}
\end{array}
\qquad
\begin{array}{ccccccc}
 & l_c & r_c & l_u & r_u & l_d & r_d \\
l_c r_c & R_{l_c}^{l_c r_c} & R_{r_c}^{l_c r_c} & \cdot & \cdot & \cdot & \cdot \\
l_u r_c & \cdot & R_{r_c}^{l_u r_c} & R_{l_u}^{l_u r_c} & \cdot & \cdot & \cdot \\
l_u r_u & \cdot & \cdot & R_{l_u}^{l_u r_u} & R_{r_u}^{l_u r_u} & \cdot & \cdot \\
l_d r_u & \cdot & \cdot & \cdot & R_{r_u}^{l_d r_u} & R_{l_d}^{l_d r_u} & \cdot \\
l_d r_d & \cdot & \cdot & \cdot & \cdot & R_{l_d}^{l_d r_d} & R_{r_d}^{l_d r_d} \\
l_c r_d & R_{l_c}^{l_c r_d} & \cdot & \cdot & \cdot & \cdot & R_{r_d}^{l_c r_d}
\end{array}
$$

Fig. 3. Examples of cyclic subsystems of rank 4 and 6.

Secondly, in addition to the definition of the quantities as presented above, there are several interesting systems produced by redefining these quantities.[1] From the description of the double-detection paradigm, one can argue, e.g., that

[1] There are also several uninteresting ways to construct systems of measurements for the conditions and measurements in this experiment. Examples of how to construct them and why they are not interesting may be found in Ref. [7]

the center location may be viewed as a signal to be detected, with either of the two off-center locations being treated as absence of the signal. This way of looking at the stimuli induces the following definition of the properties to be measured:

- l_c: a dot is presented in the center of the left circle;
- r_c: a dot is presented in the center of the right circle;
- l_{ud}: a dot is presented off-center in the left circle;
- r_{ud}: a dot is presented off-center in the right circle.

Analogously one could also consider l_{cu}, l_{cd}, r_{cu}, r_{cd}, as properties to be measured in appropriately chosen contexts,

Another way of dealing with our data is to consider the locations of the dots as properties to be measured (by responses attributing to them to a left or to a right circle). For instance, a pair of properties can be chosen as

- c: a dot is presented in the center of a circle; and
- ud: a dot is presented off the center of a circle.

A systematic application of both of these redefinitions leads to also consider quantities l_{cu}, l_{cd}, r_{cu}, r_{cd}, u, cd, d, and cu with the analogous interpretations. In this way, six systems of rank 2 and 27 systems of rank 4 may be constructed. Thus, we shall consider systems with the structures depicted by the matrices in Figures 4, 5, and 6.

$$\boxed{\begin{array}{ccc} & x & y \\ l_x r_y & R_x^{l_x r_y} & R_y^{l_x r_y} \\ l_y r_x & R_x^{l_y r_x} & R_y^{l_y r_x} \end{array}}$$

Fig. 4. Rank 2 systems structure where (x, y) is any of $(c, ud), (cu, d), (cd, u), (c, u), (c, d), (u, d)$.

$$\boxed{\begin{array}{ccccc} & l_x & r_x & l_y & r_y \\ l_x r_x & R_{l_x}^{l_x r_x} & R_{r_x}^{l_x r_x} & \cdot & \cdot \\ l_y r_x & \cdot & R_{r_x}^{l_y r_x} & R_{l_y}^{l_y r_x} & \cdot \\ l_y r_y & \cdot & \cdot & R_{l_y}^{l_y r_y} & R_{r_y}^{l_y r_y} \\ l_x r_y & R_{l_x}^{l_x r_y} & \cdot & \cdot & R_{r_y}^{l_x r_y} \end{array}}$$

Fig. 5. Rank 4 systems structure where (l_x, l_y) is any of $(l_c, l_{ud}), (l_{cu}, l_d), (l_{cd}, l_u), (l_c, l_u), (l_c, l_d), (l_u, l_d)$, and (r_x, r_y) is any of $\{(r_c, r_{ud}), (r_{cu}, r_d), (r_{cd}, r_u), (r_c, r_u), (r_c, r_d), (r_u, r_d)\}$.

$$
\begin{array}{c|cccccc}
 & l_x & r_x & l_y & r_y & l_z & r_z \\
\hline
l_x r_x & R^{l_x r_x}_{l_x} & R^{l_x r_x}_{r_x} & \cdot & \cdot & \cdot & \cdot \\
l_y r_x & \cdot & R^{l_y r_x}_{r_x} & R^{l_y r_x}_{l_y} & \cdot & \cdot & \cdot \\
l_y r_y & \cdot & \cdot & R^{l_y r_y}_{l_y} & R^{l_y r_y}_{r_y} & \cdot & \cdot \\
l_z r_y & \cdot & \cdot & \cdot & R^{l_z r_y}_{r_y} & R^{l_z r_y}_{l_z} & \cdot \\
l_z r_z & \cdot & \cdot & \cdot & \cdot & R^{l_z r_z}_{l_z} & R^{l_z r_z}_{r_z} \\
l_x r_z & R^{l_x r_z}_{l_x} & \cdot & \cdot & \cdot & \cdot & R^{l_x r_z}_{r_z}
\end{array}
$$

Fig. 6. Rank 6 systems structure where (x, y, z) is any of $(c, u, d), (c, d, u), (u, c, d), (d, c, u), (u, d, c), (d, u, c)$.

5 Results

5.1 Results for Cyclic Subsystems

Table 2 presents the individual data for all of the expectations used in the calculations of all subsystems. Note that the statistics associated with the redefined quantities are obtained by an apropriate linear combination of those in Table 2 with weights proportional to the number of trials of the combined conditions.

Table 2. Individual level data

	$P1$			$P2$			$P3$		
l r	$\langle R_l^{lr}\rangle$	$\langle R_r^{lr}\rangle$	$\langle R_l^{lr} R_r^{lr}\rangle$	$\langle R_l^{lr}\rangle$	$\langle R_r^{lr}\rangle$	$\langle R_l^{lr} R_r^{lr}\rangle$	$\langle R_l^{lr}\rangle$	$\langle R_r^{lr}\rangle$	$\langle R_l^{lr} R_r^{lr}\rangle$
l_c r_c	0.4349	0.2730	0.4825	0.7317	0.5683	0.3984	0.3582	0.1946	−0.0913
l_c r_u	0.6190	−0.5397	−0.2095	0.7016	−0.0825	−0.2413	0.6762	−0.8508	−0.6159
l_c r_d	−0.1873	0.2698	0.4095	0.8857	−0.8635	−0.7937	0.3937	−0.3524	−0.3429
l_u r_c	−0.5048	0.1175	0.2254	−0.2063	0.5238	−0.5302	−0.7302	0.6603	−0.5683
l_u r_u	0.0476	−0.0286	0.4794	0.1111	0.1683	0.2190	−0.4904	−0.6624	0.4459
l_u r_d	−0.8476	−0.0857	0.1619	0.2254	−0.7778	−0.4222	−0.7643	−0.2166	0.0446
l_d r_c	0.5873	−0.3937	−0.0825	−0.6667	0.7810	−0.5238	−0.4159	0.3429	−0.4762
l_d r_u	0.5619	−0.9111	−0.5365	−0.7333	0.2635	−0.4286	−0.2508	−0.7079	0.0095
l_d r_d	0.5111	0.3016	0.4730	−0.5175	−0.5937	0.5810	−0.3079	−0.1746	0.0413

Table 3 presents the values of ΛC, ICC, and ΔC calculated for each participant and each of the rank 6 cyclic subsystems. Table 4 presents the respective values for each of the rank 4 cyclic subsystems. For all participants, the subsystems are noncontextual.

5.2 Results for Cyclic Systems with Redefined Quantities

Table 5 presents the values of ΛC, ICC, and ΔC calculated for each participant for each of the rank 2 cyclic systems, and Table 6 shows those for the rank 4 cyclic systems. Note that for participant $P3$, two of the rank 2 systems, those

Table 3. Contextuality cyclic subsystems of rank 6

System	$P1$			$P2$			$P3$		
$(l_x, l_y, l_z), (r_x, r_y, r_z)$	ΛC	ICC	ΔC	ΛC	ICC	ΔC	ΛC	ICC	ΔC
$(l_c, l_d, l_u), (r_d, r_u, r_c)$	1.6254	2.4127	-4.7873	2.4571	1.3714	-2.9143	2.0382	1.0779	-3.0397
$(l_d, l_c, l_u), (r_c, r_d, r_u)$	1.7143	2.4889	-4.7746	2.4508	1.4286	-2.9778	2.4078	1.3138	-2.9060
$(l_d, l_u, l_c), (r_c, r_u, r_d)$	1.9873	3.4476	-5.4603	2.3476	0.7286	-2.3810	1.4104	0.8040	-3.3936
$(l_c, l_u, l_d), (r_d, r_c, r_u)$	2.6063	2.2952	-3.6889	2.9508	1.0968	-2.1460	1.4991	1.0213	-3.5222
$(l_u, l_c, l_d), (r_d, r_u, r_c)$	1.7238	2.7206	-4.9968	2.3857	0.9413	-2.5556	1.7151	1.0784	-3.3633
$(l_u, l_d, l_c), (r_c, r_d, r_u)$	1.7651	1.4921	-3.7270	2.1190	1.2524	-3.1333	1.3708	1.0598	-3.6890

Table 4. Contextuality cyclic subsystems of rank 4

System	$P1$			$P2$			$P3$		
$(l_x, l_y), (r_x, r_y)$	ΛC	ICC	ΔC	ΛC	ICC	ΔC	ΛC	ICC	ΔC
$(l_c, l_u), (r_c, r_u)$	1.3968	1.4032	-2.0063	0.9508	0.6429	-1.6921	1.7213	1.2118	-1.4904
$(l_c, l_u), (r_c, r_d)$	0.9556	1.4762	-2.5206	2.1444	0.7159	-0.5714	1.0470	0.6711	-1.6241
$(l_c, l_u), (r_u, r_d)$	1.2603	2.5683	-3.3079	1.6762	0.6349	-0.9587	1.3600	0.8806	-1.5206
$(l_c, l_d), (r_c, r_u)$	1.3111	1.2476	-1.9365	1.5921	0.6556	-1.0635	1.1929	0.7742	-1.5812
$(l_c, l_d), (r_c, r_d)$	1.4476	1.3968	-1.9492	1.5000	0.7857	-1.2857	0.9517	0.4694	-1.5177
$(l_c, l_d), (r_u, r_d)$	1.2095	1.2603	-2.0508	2.0444	1.0159	-0.9714	0.9905	0.6603	-1.6698
$(l_u, l_d), (r_c, r_u)$	1.1587	1.9714	-2.8127	1.7016	0.7365	-1.0349	1.4808	0.7678	-1.2870
$(l_u, l_d), (r_c, r_d)$	0.9429	1.3175	-2.3746	2.0571	1.0222	-0.9651	1.0478	0.5015	-1.4538
$(l_u, l_d), (r_u, r_d)$	1.6508	2.2159	-2.5651	1.2127	0.6095	-1.3968	0.5222	0.4185	-1.8963

with $(x, y) = (c, d)$ and $(x, y) = (cu, d)$, have a positive ΔC value, which might suggest that these two systems show contextuality. However, their respective confidence intervals, $\Delta C_{(cu,d)} \in (-0.267, 0.241)$ and $\Delta C_{(c,d)} \in (-0.233, 0.215)$,[2] indicate that the values are consistent with lack of contextuality.

Table 5. Contextuality cyclic systems of rank 2

System	$P1$			$P2$			$P3$		
(x,y)	ΛC	ICC	ΔC	ΛC	ICC	ΔC	ΛC	ICC	ΔC
(c, ud)	0.0286	0.5302	-0.5016	0.0095	0.1778	-0.1683	0.0429	0.0619	-0.0190
(cd, u)	0.5228	0.5947	-0.0720	0.1905	0.2286	-0.0381	0.0430	0.0631	-0.0201
(cu, d)	0.5608	0.5862	-0.0254	0.1778	0.2032	-0.0254	0.1003	0.0695	0.0308
(c, u)	0.4349	0.5365	-0.1016	0.2889	0.3016	-0.0127	0.0476	0.1365	-0.0889
(c, d)	0.4921	0.5238	-0.0317	0.2698	0.3016	-0.0317	0.1333	0.1143	0.0190
(u, d)	0.6984	0.7111	-0.0127	0.0063	0.0825	-0.0762	0.0351	0.0906	-0.0556

[2] 95% confidence intervals corrected by Bonferroni for the number of tests for ΔC values in the experiment. However, it should be noted that even uncorrected intervals covered the value 0.

Table 6. Contextuality cyclic systems of rank 4

System	$P1$			$P2$			$P3$		
$(l_x, l_y), (r_x, r_y)$	ΛC	ICC	ΔC	ΛC	ICC	ΔC	ΛC	ICC	ΔC
$(l_c, l_{ud}), (r_c, r_{ud})$	0.6556	0.7032	-2.0476	1.4556	0.5921	-1.1365	1.2281	0.7648	-1.5367
$(l_c, l_{ud}), (r_{cd}, r_u)$	0.7926	1.1228	-2.3302	0.6720	0.5238	-1.8519	1.3525	0.9192	-1.5667
$(l_c, l_{ud}), (r_{cu}, r_d)$	0.9407	1.3937	-2.4529	1.2857	0.7460	-1.4603	0.9247	0.5181	-1.5934
$(l_c, l_{ud}), (r_c, r_u)$	0.7349	0.9286	-2.1937	1.2714	0.5381	-1.2667	1.4568	0.9931	-1.5363
$(l_c, l_{ud}), (r_c, r_d)$	1.1381	1.4048	-2.2667	1.6397	0.7063	-1.0667	0.9993	0.5365	-1.5371
$(l_c, l_{ud}), (r_u, r_d)$	0.9079	1.5111	-2.6032	1.2190	0.8254	-1.6063	1.1431	0.7703	-1.6271
$(l_{cd}, l_u), (r_c, r_{ud})$	0.7841	0.4688	-1.6847	1.0423	0.6106	-1.5683	1.3443	0.7911	-1.4469
$(l_{cd}, l_u), (r_{cd}, r_u)$	1.3418	1.6402	-2.2984	1.0681	0.4804	-1.4123	1.4357	0.9428	-1.5070
$(l_{cd}, l_u), (r_{cu}, r_d)$	0.8127	1.6275	-2.8148	0.9975	0.5284	-1.5309	0.7726	0.5453	-1.7727
$(l_{cd}, l_u), (r_c, r_u)$	1.3175	1.3683	-2.0508	0.9619	0.6106	-1.6487	1.6412	1.0639	-1.4227
$(l_{cd}, l_u), (r_c, r_d)$	0.7884	1.2159	-2.4275	1.3788	0.7037	-1.3249	1.0473	0.5867	-1.5394
$(l_{cd}, l_u), (r_u, r_d)$	1.3905	2.4508	-3.0603	1.2804	0.4487	-1.1683	1.0235	0.6986	-1.6751
$(l_{cu}, l_d), (r_c, r_{ud})$	0.6212	0.9725	-2.3513	0.9153	0.6868	-1.7714	0.9903	0.4030	-1.4127
$(l_{cu}, l_d), (r_{cd}, r_u)$	1.0328	1.3848	-2.3520	0.6603	0.6145	-1.9541	0.8142	0.5372	-1.7230
$(l_{cu}, l_d), (r_{cu}, r_d)$	1.3051	1.4399	-2.1347	1.7129	0.8698	-1.1570	0.8240	0.2918	-1.4677
$(l_{cu}, l_d), (r_c, r_u)$	0.9958	1.4889	-2.4931	1.1291	0.6423	-1.5132	0.9988	0.5452	-1.5464
$(l_{cu}, l_d), (r_c, r_d)$	1.2794	1.3704	-2.0910	1.6857	0.8646	-1.1788	0.9818	0.2608	-1.2790
$(l_{cu}, l_d), (r_u, r_d)$	1.3566	1.5788	-2.2222	1.7672	0.8804	-1.1132	0.5084	0.5496	-2.0412
$(l_c, l_u), (r_c, r_{ud})$	0.9286	0.5571	-1.6286	1.5476	0.6492	-1.1016	1.3842	0.9073	-1.5231
$(l_c, l_u), (r_{cd}, r_u)$	1.3513	1.7915	-2.4402	0.9534	0.5069	-1.5534	1.6014	1.1021	-1.5007
$(l_c, l_u), (r_{cu}, r_d)$	0.8095	1.6328	-2.8233	1.6815	0.6296	-0.9481	0.8847	0.6947	-1.8101
$(l_c, l_d), (r_c, r_{ud})$	0.6333	1.1063	-2.4730	1.3635	0.6238	-1.2603	1.0723	0.6218	-1.5495
$(l_c, l_d), (r_{cd}, r_u)$	1.1016	1.1968	-2.0952	0.8265	0.7757	-1.9492	1.1043	0.7363	-1.6320
$(l_c, l_d), (r_{cu}, r_d)$	1.3683	1.3513	-1.9831	1.6815	0.8624	-1.1810	0.9647	0.4479	-1.4833
$(l_u, l_d), (r_c, r_{ud})$	0.5968	0.9143	-2.3175	1.2317	0.8127	-1.5810	1.2643	0.5585	-1.2942
$(l_u, l_d), (r_{cd}, r_u)$	1.3228	1.7608	-2.4381	1.2974	0.6180	-1.3206	1.1044	0.6240	-1.5195
$(l_u, l_d), (r_{cu}, r_d)$	1.1788	1.6169	-2.4381	1.7757	0.8847	-1.1090	0.5485	0.4365	-1.8880

6 Conclusions

The experiment presented in this paper illustrates the use of the double factorial paradigm in the search of contextuality in behavioral systems, namely in the responses of human observers in a double-detection task. This paradigm provides the closest analogue in psychophysical research to the Alice-Bob EPR/Bohm paradigm.

We have found that for the participants in the study there was no evidence of contextuality in their responses. These results add to the existing evidence that points towards lack of contextuality in psychology (cf. Ref. [13].)

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