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Preface

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Contextuality and probability in quantum mechanics and beyond: a preface

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This special issue is loosely based on the Purdue Winer Memorial Lectures 2018, an interdisciplinary meeting held at Purdue University in November 2018. I take this opportunity to express my gratitude to the late Benjamin Winer, a prominent psychometrician who worked at Purdue in 1954–1984, for leaving a legacy that funded this and other Winer Memorial Lectures. My gratitude also goes to Ben Winer's sister Sylvia VerMeer, who generously added to this legacy before her passing away in 2008. Purdue Winer Memorial Lectures have been held with approximate regularity of once every 2 years since my organizing the first such meeting in 2002. Each of these meetings had a broadly defined topic, and in 2018 it was 'Probability and contextuality'.

It would not be fair, however, to relate the present collection of papers to only this one meeting. Most of the contributors to this issue were also participants in the annual workshop that I have been holding since 2017 in Prague, Czech Republic, under the name 'Quantum contextuality in quantum mechanics and beyond', financed by Purdue University. I should also mention the prominent role of the Purdue Winer Memorial Lectures 2014, 'Contextuality from quantum physics to psychology', based on which World Scientific published a book of chapters under the same name [1]. This was arguably the first interdisciplinary meeting entirely dedicated to contextuality. I would like to thank Víctor H. Cervantes, Ru Zhang, Lacey Perry and Maria Kon, doctoral students at Purdue University, for their invaluable help in organizing and running some of these meetings.

This special issue would not be possible without the work and expertise of my fellow coeditors: Samson Abramsky, Adán Cabello and Paweł Kurzyński. They also served as members of the scientific committees of the Purdue Winer Memorial Lectures 2018 and the Prague contextuality workshops. In addition, I should thank my fellow coeditors for critically reading and commenting

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on this preface. With their consent, it reflects my personal perspective on the special issue, almost certainly differing from theirs in many ways.

The interdisciplinary aspect of the present collection is prominent, as the 'beyond' in its title indicates. The contributors to this issue include physicists, mathematicians, philosophers, computer scientists and psychologists. This may be a cause for concern with some readers. The grammatical derivatives of 'context' are common in psychology, linguistic and literary critique, one might argue, but surely these words are used in different meanings in these different fields, and none may be related to contextuality in quantum theory. Could all similarities between the latter and the 'beyond' be superficial or metaphorical? Such concerns are amply justified, as one can find plenty of writing whose authors emphasize superficial analogies over analytic distinctions. One can even find some instances thereof in the above-mentioned book of chapters [1]. The lesson one can learn from this is that one should look for the commonality of ideas and mathematical developments rather than similarity of words. One would find then that contextuality analysis of the same mathematical structure as in quantum mechanics can be reached by the internal logic of research in fields outside quantum physics.

A good illustration is provided by the story of the convergence to essentially the same notion of contextuality of two completely unrelated conceptual systems and research lines, in quantum physics and in psychology. In the matrix below, the q's are certain entities (things, properties), the c's are contexts defined by what two entities are paired, and a star symbol in a cell (q_i, c^j) indicates that the property q_i is measured in the context c^j. Assuming that each measurement can have one of two possible values, denoted ±1, this matrix represents what in the theory of contextuality is called a cyclic system (of rank 4).

⋆	⋆			c^1
	⋆	⋆		c^2
		⋆	⋆	c^3
⋆			⋆	c^4
q_1	q_2	q_3	q_4	$\mathcal{R}$

(1)

In quantum physics, it can be realized in a variety of ways, but its most celebrated use is in representing the 'Alice-Bob' scenario with two entangled spin-1/2 particles [2–4]. In this scenario, q_1 and q_3 are axes chosen by Alice for measuring her particle's spins, q_2 and q_4 are axes chosen by Bob for measuring his particle's spins, and the contexts are defined by which of their axes Alice and Bob choose simultaneously in a given experiment. The contextuality analysis of this system was initially presented as a response to the question that did not involve words derived from 'context': is there a random variable λ such that if q_i is measured in context c^j, the result can be presented as a function of q_i and of λ? The affirmative answer to this question is known as the Local Hidden Variable Theory, although a better term would have been something like Hidden Variable Theory with Context-Independent (or Local) Mappings. The history of this question is well known, dating back to the 1935 discussion involving Einsten [5] and Bohr [6]. It is also well known that quantum physics allows for systems for which the answer to this question is negative—in which case we say that the system is Bell-nonlocal, or, more generally, contextual.

In psychology, about the same time as the CHSH inequalities [4] and the Kochen–Specker theorem [7] were published, Saul Sternberg [8] posed a question that one would hardly suspect of being related to the Local Hidden Variable Theory. Psychology is largely about responses of organisms or persons to inputs, generically called stimuli, and one generally deals with several stimuli evoking several responses. Sternberg pointed out that it was important to know which of several stimuli a given response responds to. This problem has since become known as that of selectiveness of influences. As a simple case, consider responses of an organism to two stimuli S_x and S_y (say, the size and the brightness of a visual object), each of which could have one of two values that can be combined in four possible ways (e.g. large or small size combined with high or low brightness). Denote by q_1, q_3 the two values of S_x, and by q_2, q_4 the values of

S_y. The experiment can then be described by matrix (1), except that the star symbol in a cell (q_i, c^j) indicates now that the stimulus q_i has been responded to in the context c^j (i.e. when q_i was presented together with another stimulus, $q_{i'}$, whose pairing with q_i forms the context c^j). Sternberg's question can now be formulated thus: how to determine that the response to q_i in context c^j is influenced by q_i alone rather than also influenced by $q_{i'}$? For a lengthy period of time, the only necessary condition for the selectiveness of influences was 'marginal selectivity' [9]—the independence of the distribution of the responses to q_i of the context in which it is recorded. This is the analogue of the 'no-signalling' or 'no-disturbance' condition in quantum physics (also known by a variety of other names). Only in 2003, in [10], it was made clear that Sternberg's question is about the existence of a random variable λ such that the response to q_i in any context in which it was responded to is a function of λ and of q_i. In other words, the hypothesis that the response of the organism to S_x does not depend on S_y, and vice versa, is equivalent to the Local Hidden Variable Theory. Unlike in quantum theory, here, the equivalence of the deterministic and probabilistic versions of the Local Hidden Variable Theory was made clear from the outset.

The development of the research on selectiveness of influences in psychology went on blissfully unaware of any connections with quantum physics for several years, during which some of the necessary conditions and criteria of (non)contextuality developed in quantum physics were rediscovered and formulated as necessary conditions and criteria of the selectiveness of influences. Thus, Lawrence Landau's inequality [11], nowadays often considered a criterion of the compliance of matrix (1) with quantum mechanical restrictions, has been formulated and proved in a completely different way under the name of 'cosphericity condition' [12]. The 'joint distribution criterion' [13] was a streamlined formulation of the demonstrations by Suppes & Zanotti [14] and by Fine [15] that the Local Hidden Variable Theory is equivalent to the hypothesis that the star symbols in a matrix like (1) can be replaced with jointly distributed random variables labelled by the respective q's (irrespective of the c's) [15]. It was not until 2012 that the development of the theory of selective influences was merged with contextuality theory in quantum physics, as just another application of essentially the same mathematical construction [16].

Other examples of non-physical applications of the contextuality theory can be found in the work of Abramsky *et al*. [17,18] (see Abramsky and Caru's paper in this collection). This prominently includes contextuality analysis of record-sharing databases. Unlike in the study of selective influences, the commonalities with quantum contextuality here were established from the outset, but they too grew from the internal logic of the analysis of databases rather than from any terminological similarities.

This should be sufficient to justify the 'beyond' in the title of this collection of papers. I also need, however, to comment on the 'and probability' part of the title. On the one hand, this serves to emphasize the fact that contextuality analysis in quantum physics and in many non-physical applications is fundamentally probabilistic. For instance, in my favourite approach to contextuality, called Contextuality-by-Default, theory of contextuality is essentially coextensive with the theory of random variables. On the other hand, the title says 'contextuality and probability' rather than 'probabilistic contextuality'. The conjunction 'and' puts a conceptual distance between the two notions. For one thing, this serves as an acknowledgment of the possibility that contextuality analysis can apply to non-probabilistic scenarios too, as it is done in the sheaf-theoretic approach to contextuality introduced by Abramsky & Brandenburger [17]. For another, it serves to acknowledge the prominent research in the foundations of quantum physics and abstract quantum theory (understood as a probability theory detached from specific physical applications), where contextuality plays the role of a guiding light. This line of research is represented by recent work by Cabello *et al*. [19–21], including Cabello's paper in this collection.

I will refrain from characterizing or even describing all the papers included in this special issue. Looking through the abstracts of the papers will be more informative for this purpose. I should only mention that the contributions to this collection by no means present a unifying approach to contextuality and probability. Thus, Robert Griffiths's treatment of contextuality in terms of counterfactual assignment of values contradicts the position of several other authors. Sergey Rashkovskiy and Andrei Khrennikov see contextuality in the violations of the classical formula of total probability, which is only indirectly related to the traditional understanding derived from the Kochen–Specker theorem and Bell-nonlocality studies [2–4,7,15]. Some of the contributors would probably agree that contextuality is about the possibility or impossibility of imposing a global structure on a set in a way that agrees with the structure of various subsets of the set—but they may disagree about precise definitions of the structure and of the subsets. Nevertheless, this special issue presents a very good approximation to the state of the art in our understanding of contextuality.

References

1. Dzhafarov EN, Jordan JS, Zhang R, Cervantes VH (eds). 2015 *Contextuality from quantum physics to psychology*. Hackensack, NJ: World Scientific.
2. Bell J. 1964 On the Einstein-Podolsky-Rosen paradox. *Physics* **1**, 195–200. (doi:10.1103/PhysicsPhysiqueFizika.1.195)
3. Bell J. 1966 On the problem of hidden variables in quantum mechanics. *Rev. Mod. Phys.* **38**, 447–453. (doi:10.1103/RevModPhys.38.447)
4. Clauser JF, Horne MA, Shimony A, Holt RA. 1969 Proposed experiment to test local hidden-variable theories. *Phys. Rev. Lett.* **23**, 880–884. (doi:10.1103/PhysRevLett.23.880)
5. Einstein A, Podolsky B, Rosen N. 1935 Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777–780. (doi:10.1103/PhysRev.47.777)
6. Bohr N. 1935 Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.* **48**, 696–702. (doi:10.1103/PhysRev.48.696)
7. Kochen S, Specker EP. 1967 The problem of hidden variables in quantum mechanics. *J. Math. Mech.* **17**, 59–87.
8. Sternberg S. 1969 The discovery of processing stages: extensions of Donders method. In *Attention and performance II* (ed WG Koster). Acta Psych., no. 30, pp. 276–315.
9. Townsend JT, Schweickert R. 1989 Toward the trichotomy method of reaction times: laying the foundation of stochastic mental networks. *J. Math. Psychol.* **33**, 309–327. (doi:10.1016/0022-2496(89)90012-6)
10. Dzhafarov EN. 2003 Selective influence through conditional independence. *Psychometrika* **68**, 7–25. (doi:10.1007/BF02296650)
11. Landau L. 1988 Empirical two-point correlation functions. *Found. Phys.* **18**, 449–460. (doi:10.1007/BF00732549)
12. Kujala JV, Dzhafarov EN. 2008 Testing for selectivity in the dependence of random variables on external factors. *J. Math. Psychol.* **52**, 128–144. (doi:10.1016/j.jmp.2008.01.008)
13. Dzhafarov EN, Kujala JV. 2010 The joint distribution criterion and the distance tests for selective probabilistic causality. *Front. Quant. Psychol. Meas.* **1**, 151. (doi:10.3389/fpsyg.2010.00151)
14. Suppes P, Zanotti M. 1981 When are probabilistic explanations possible? *Synthese* **48**, 191–199. (doi:10.1007/BF01063886)
15. Fine A. 1982 Hidden variables, joint probability, and the Bell inequalities. *J. Math. Phys.* **23**, 1306–1310. (doi:10.1063/1.525514)
16. Dzhafarov EN, Kujala JV. 2012 Selectivity in probabilistic causality: where psychology runs into quantum physics. *J. Math. Psychol.* **56**, 54–63. (doi:10.1016/j.jmp.2011.12.003)
17. Abramsky S, Brandenburger A. 2011 The sheaf-theoretic structure of non-locality and contextuality. *New J. Phys.* **13**, 113036. (doi:10.1088/1367-2630/13/11/113036)
18. Abramsky S, Barbosa RS, Mansfield S. 2017 The contextual fraction as a measure of contextuality. *Phys. Rev. Lett.* **119**, 050504. (doi:10.1103/PhysRevLett.119.050504)
19. Cabello A. 2013 Simple explanation of the quantum violation of a fundamental inequality. *Phys. Rev. Lett.* **110**, 060402. (doi:10.1103/PhysRevLett.110.060402)
20. Cabello A. 2019 Quantum correlations from fundamental principles. (http://arxiv.org/abs/1801.06347).
21. Chiribella G, Cabello A, Kleinmann M, Müller MP. 2019 General Bayesian theories and the emergence of the exclusivity principle. (http://arxiv.org/abs/1901.11412).